

Nonstationary Seismic Reflectivity Inversion Based on Prior-engaged Mixed Dimensional Deep Learning Method

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SAIG Annual Meeting



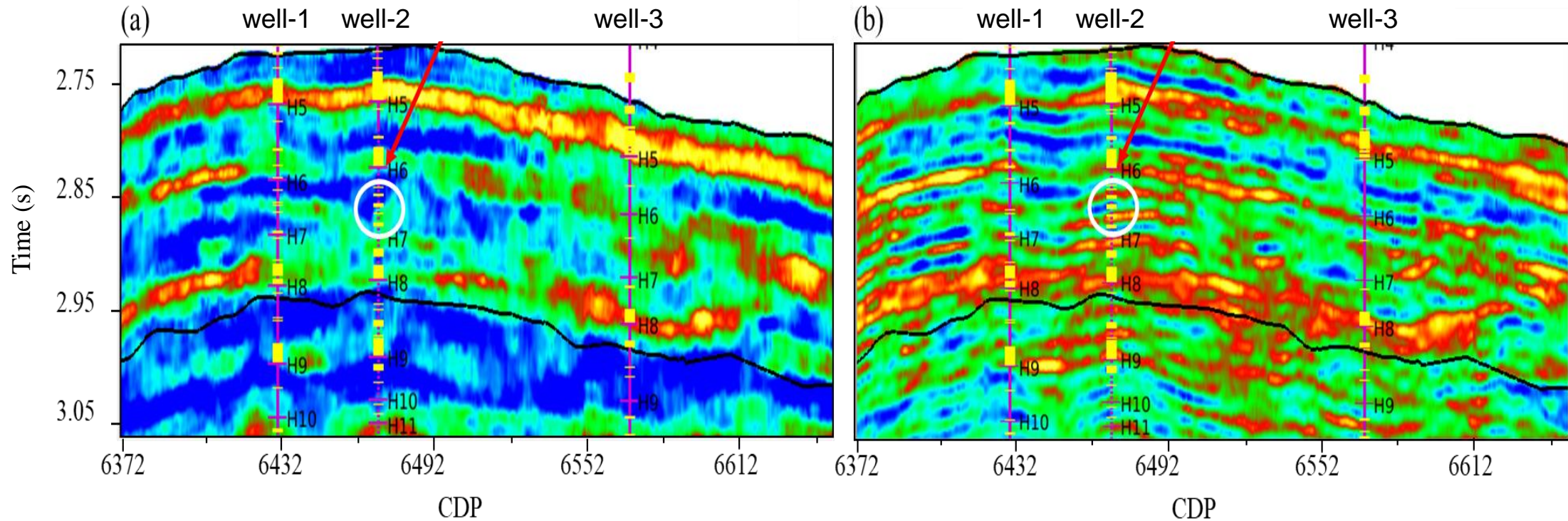
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Outline

- Introduction
- Theory
- Examples
- Conclusions
- Acknowledgments

Introduction

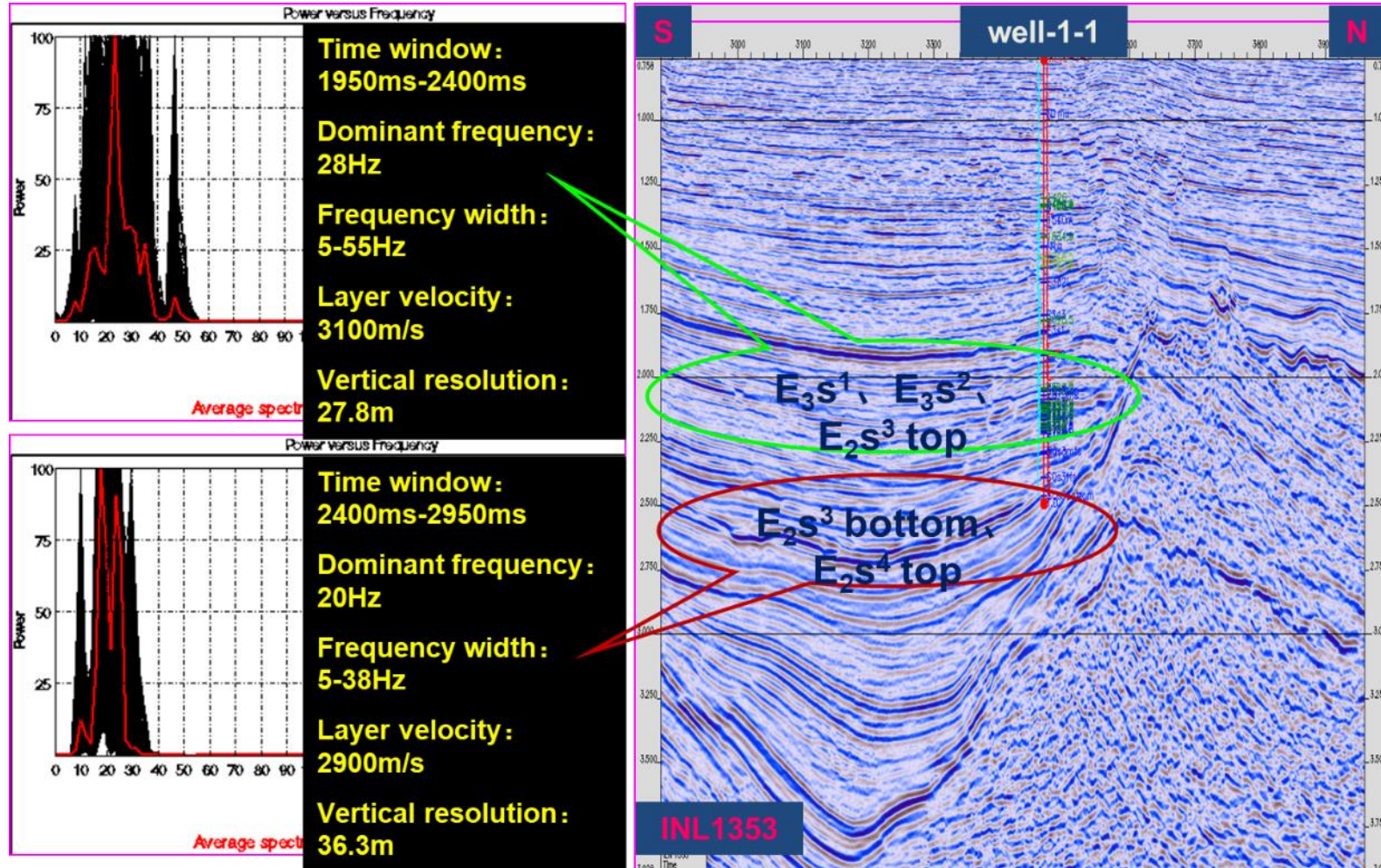
□ Significance of seismic reflectivity inversion



✓ Improving the resolution of the seismic data is conducive to obtaining the high-resolution impedance.

Introduction

□ Field data case



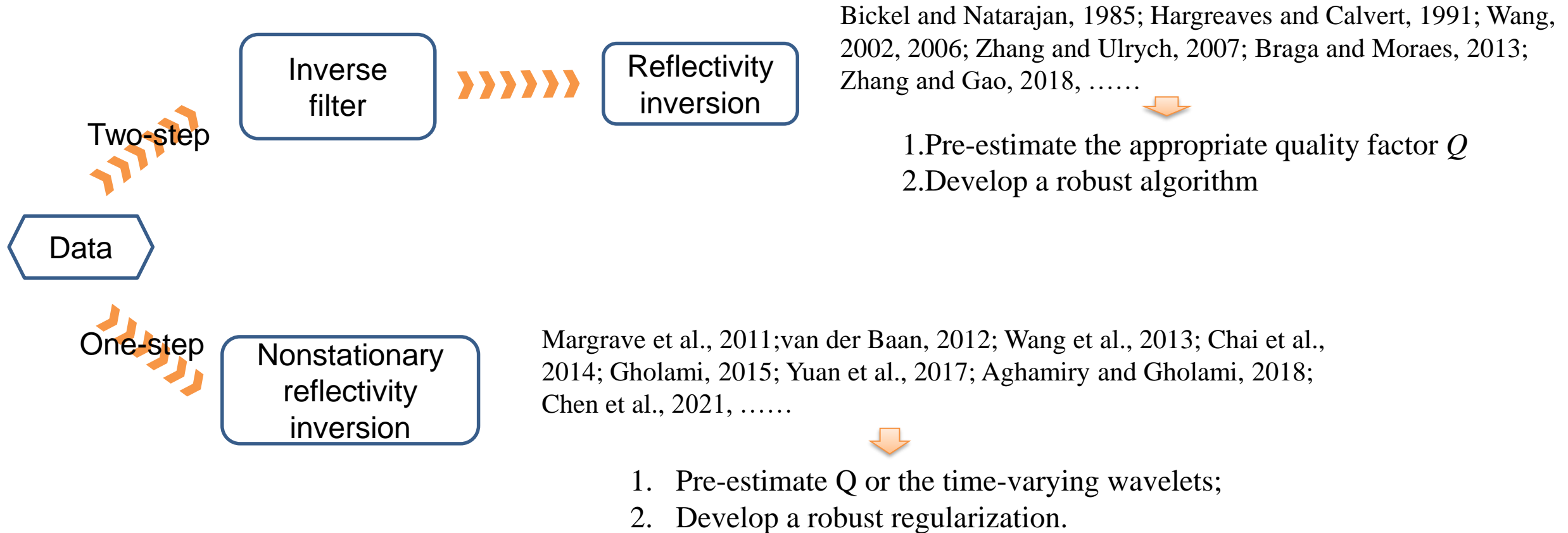
The field data is usually nonstationary because of the heterogeneous, anisotropic, and anelastic mediums.

We focus on the anelastic attenuation and dispersion, i.e., the wavelet is time-varying with amplitude attenuation and phase dispersion.

Introduction

□ Related work

There are two kinds of methods:



Introduction

□ Mathematical framework of nonstationary reflectivity inversion

$$\tilde{r} = \arg \min_r L(r, y) + \Phi(r)$$

---- y is the observed seismic data

----- r is the estimated high-resolution result

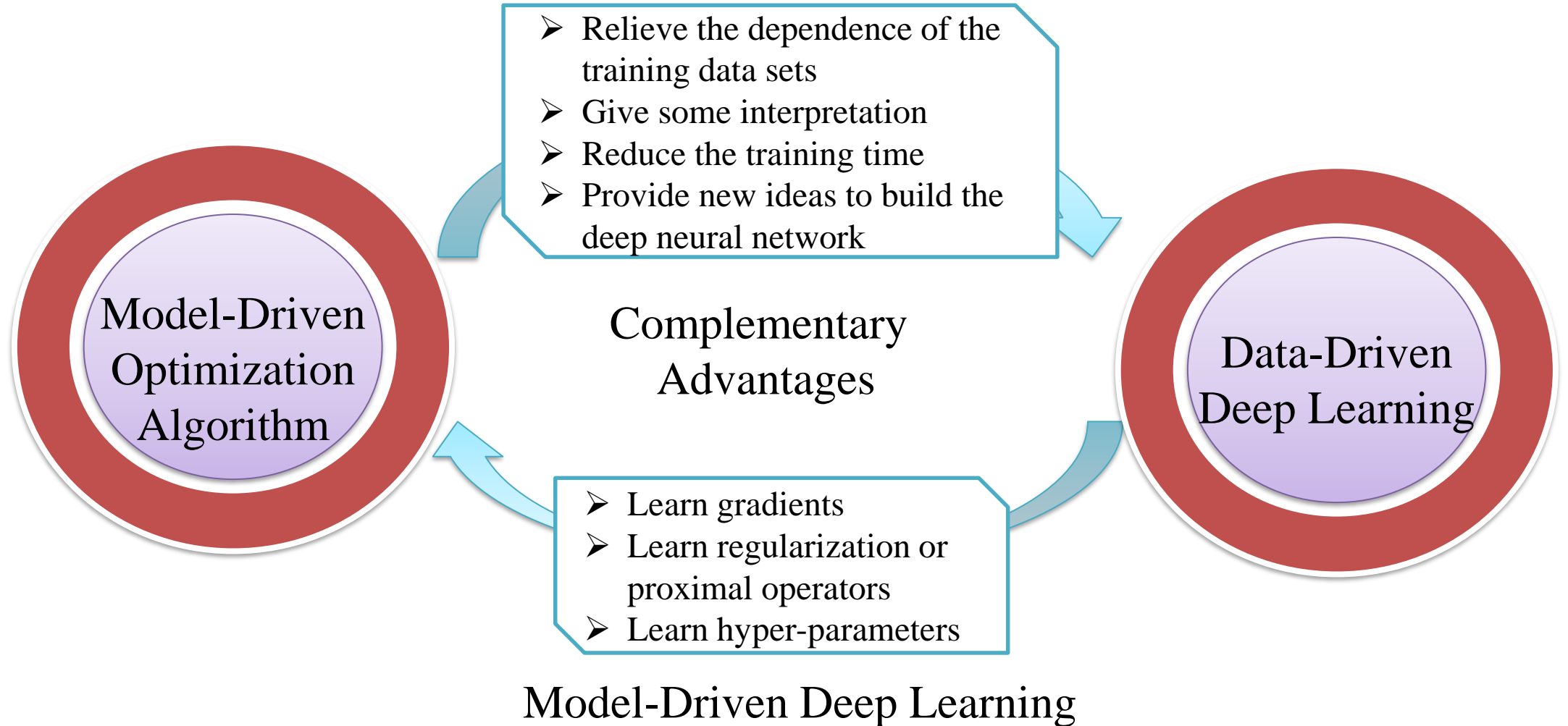
----- $L(\cdot)$ is the loss function

----- $\Phi(\cdot)$ is the penalty term

- Key challenges:**
1. estimating the time-varying wavelets or Q values;
 2. pre-determining the regularization terms and parameters;
 3. depending on the initial values;
 4. low computational efficiency.

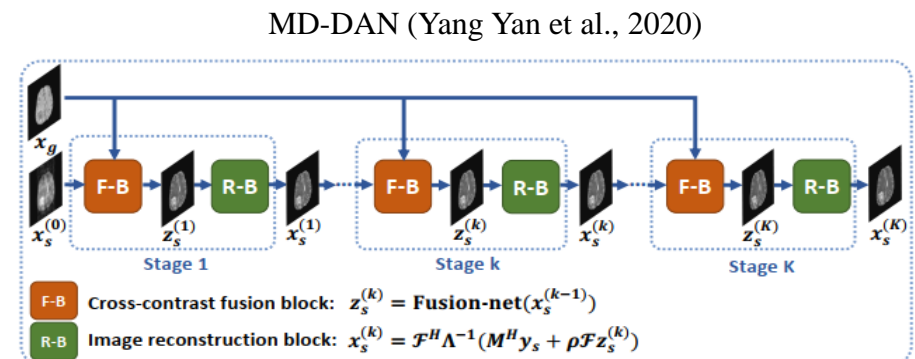
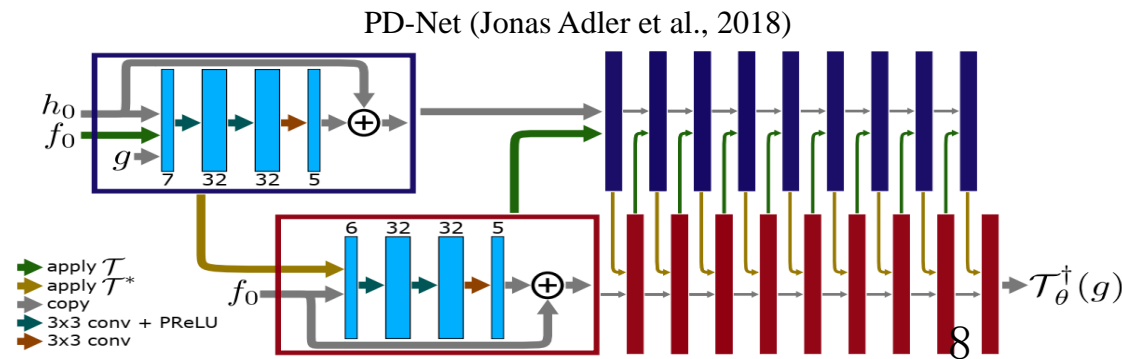
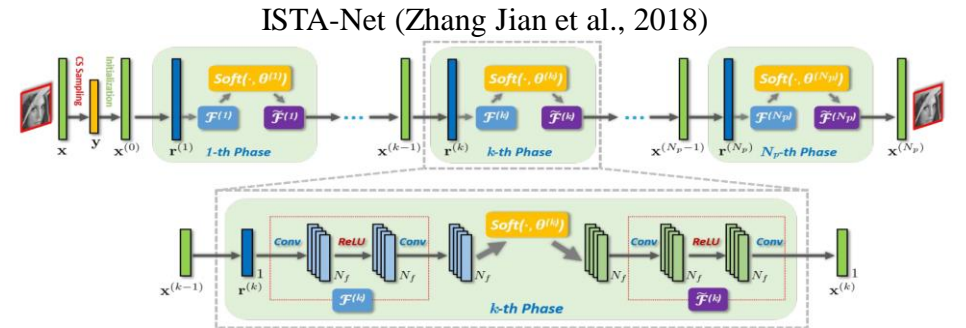
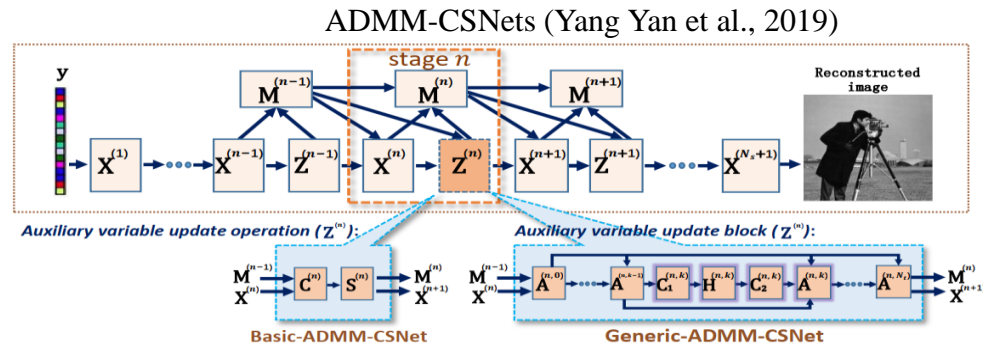
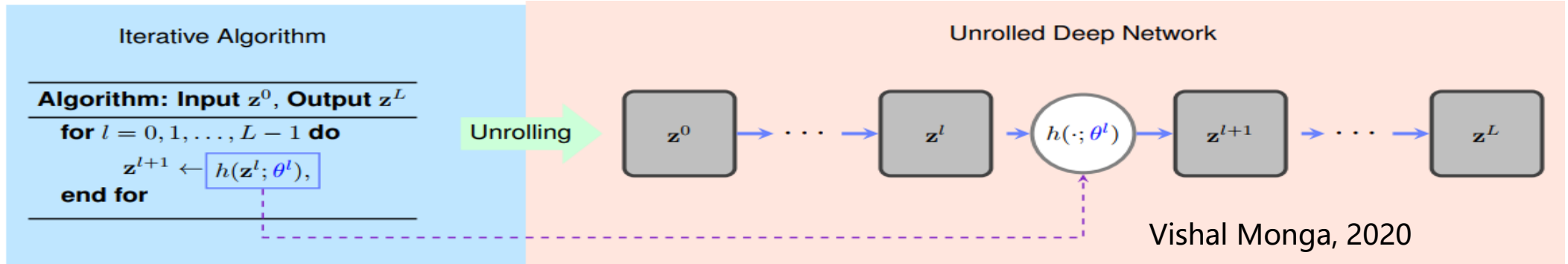
Introduction

□ How to apply deep learning to solve the seismic inverse problems?



Introduction

□ Model-Driven Deep Learning



Theory

Theory

$s(t)$: seismic trace
 $r(t)$: reflectivity
 $n(t)$: random noise
 $w(t)$: source wavelet
 $a(t, \tau)$: impulse response of the attenuation process

□ Nonstationary convolution model

According to Margrave et al. (2011), nonstationary seismic trace can be modeled by

$$s(t) = w(t) * a(t, \tau) \odot r(t) + n(t) \quad (1)$$

Based on the Kolsky-Futterman Q model, $a(t, \tau)$ is defined as:

$$a(t, \tau) = \int_{-\infty}^{\infty} \alpha(t, f) e^{2\pi i f \tau} df = \int_{-\infty}^{\infty} e^{-\frac{\pi f t}{Q}} e^{i \frac{1}{\pi} \ln\left(\frac{f}{f_r}\right) \frac{2\pi f t}{Q}} e^{2\pi i f \tau} df \quad (2)$$

where $\alpha(t, f)$ is the attenuation function, including amplitude attenuation and phase dispersion.

Theory

$w(t, \tau)$: time-varying wavelet
 $\mathbf{s}, \mathbf{r}, \mathbf{n}$: vectors of seismic trace, reflectivity and noise
 Ω_j : the j th window
 \mathbf{w}_j : wavelet in the j th window

□ Nonstationary convolution model

Rewrite equation (1) as:

$$s(t) = w(t, \tau) \odot r(t) + n(t) \quad (3)$$

where the wavelet is time-varying during propagation. This increases the instability and uncertainty of the inversion solution. Here, it is simplified as:

$$\mathbf{s} \approx \sum_{j=1}^B \mathbf{w}_j * [\mathbf{r}\Omega_j] + \mathbf{n} \quad (4)$$

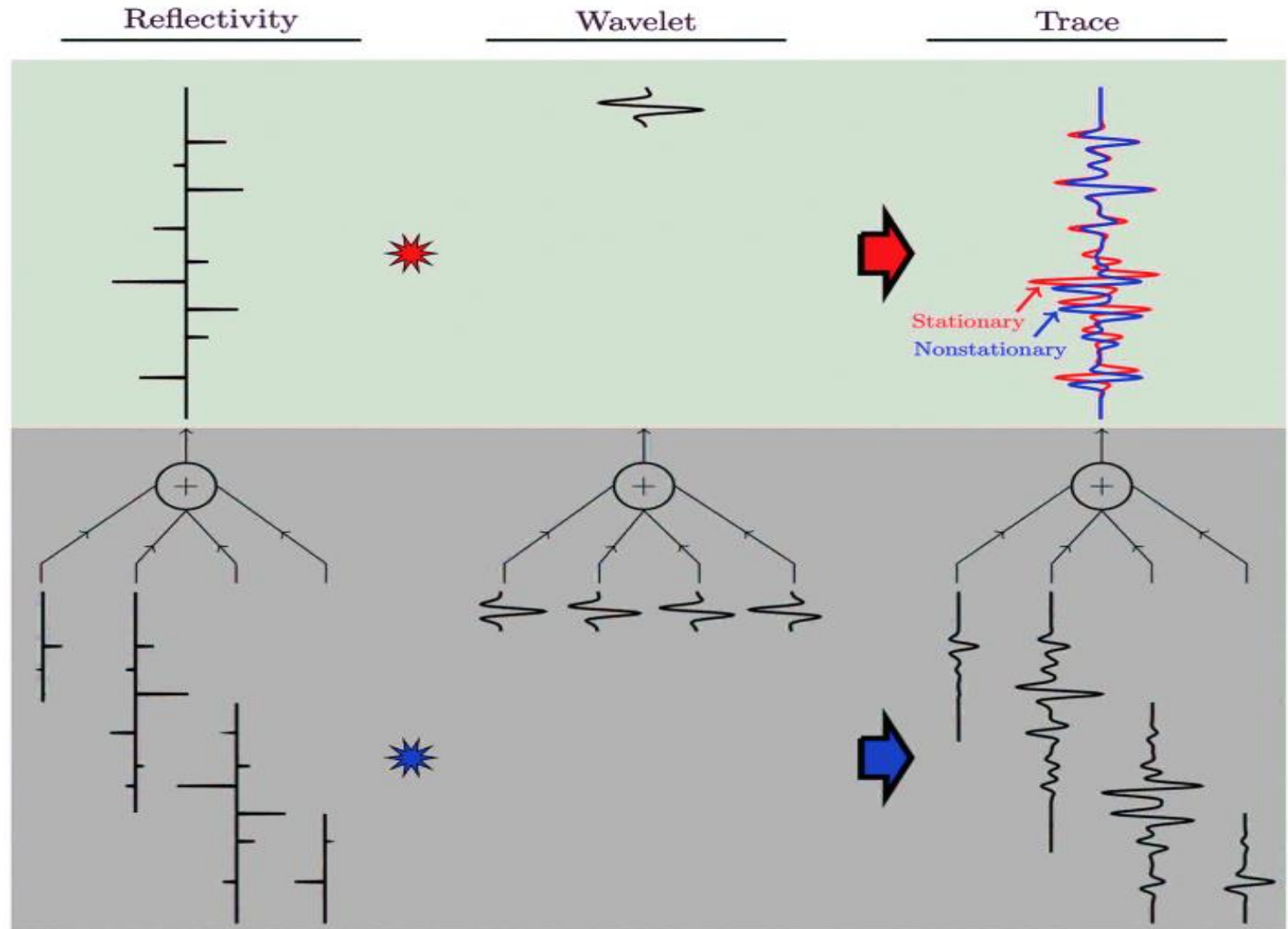
Using a set of windows to segment the reflectivity and treating that the wavelet at each window is stationary. In this case, attenuation function $\alpha(t, f)$ is considered to be slowly changing relative to the windows.

Theory

□ Nonstationary convolution model

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$$(4) \quad \mathbf{s} \approx \sum_{j=1}^B \mathbf{w}_j * [\mathbf{r}\Omega_j] + \mathbf{n}$$



Theory

□ Seismic reflectivity inversion-SRI framework

To invert the reflectivity from equation (4), the following cost function is built as:

$$J = \min_{\mathbf{r}, \mathbf{w}} \frac{1}{2} \left\| \mathbf{s} - \sum_{j=1}^B \mathbf{w}_j * [\mathbf{r}\Omega_j] \right\|_2^2 + \lambda \Psi(\mathbf{r}) + \mu \Phi(\mathbf{w}) \quad (5)$$

There are some limitations:

- Requiring to set the initial values for the seismic wavelets;
- Optimization algorithms are usually computationally demanding;
- Pre-determining the regularization terms and some sensitive parameters.

Theory

□ PMDDL M: Deep learning based nonstationary SRI

To alleviate the above limitations, we use the convolutional neural network to replace the gradient components. To derive the deep neural network, we start from the optimization of equation (5).

$$J = \min_{\mathbf{r}, \mathbf{w}} \frac{1}{2} \left\| \mathbf{s} - \sum_{j=1}^B \mathbf{w}_j * [\mathbf{r}\Omega_j] \right\|_2^2 + \lambda \Psi(\mathbf{r}) + \mu \Phi(\mathbf{w}) \quad (5)$$

 splitting

$$J_1 = \min_{\mathbf{w}} \frac{1}{2} \left\| \mathbf{s} - \sum_{j=1}^B \mathbf{w}_j * [\mathbf{r}\Omega_j] \right\|_2^2 + \mu \Phi(\mathbf{w}) \quad (6)$$

$$J_2 = \min_{\mathbf{r}} \frac{1}{2} \left\| \mathbf{s} - \sum_{j=1}^B \mathbf{w}_j * [\mathbf{r}\Omega_j] \right\|_2^2 + \lambda \Psi(\mathbf{r}) \quad (7)$$

Theory

□ PMDDLMM: Deep learning based nonstationary SRI

For each sub-problems, we use half-quadratic splitting algorithm to solve them, and then obtain the following solutions:

$$\mathbf{x}^k = \text{Concat}\left\{F^H \frac{(\bar{\mathbf{r}}_j^{k-1})^* \cdot \bar{\mathbf{s}}_j + 2\xi^k \bar{\mathbf{w}}_j^{k-1}}{(\bar{\mathbf{r}}_j^{k-1})^* \cdot \bar{\mathbf{r}}_j^{k-1} + 2\xi^k}\right\} \quad (8-1)$$

$$\mathbf{w}^k = \mathbf{w}^{k-1} - \nu^k (\mu^k \nabla \Phi(\mathbf{w}^{k-1}) + 2\xi^k (\mathbf{w}^{k-1} - \mathbf{x}^k)) \quad (8-2)$$

$$\mathbf{z}^k = \sum_{j=1}^B F^H \frac{(\bar{\mathbf{w}}_j^k)^* \cdot \bar{\mathbf{s}}_j + 2\beta^k \bar{\mathbf{r}}_j^{k-1}}{(\bar{\mathbf{w}}_j^k)^* \cdot \bar{\mathbf{w}}_j^k + 2\beta^k} \quad (8-3)$$

$$\mathbf{r}^k = \mathbf{r}^{k-1} - \zeta^k (\lambda^k \nabla \Psi(\mathbf{r}^{k-1}) + 2\beta^k (\mathbf{r}^{k-1} - \mathbf{z}^k)) \quad (8-4)$$

Theory

□ PMDDL: Deep learning based nonstationary SRI

Using CNN to replace the gradients components:

(8)

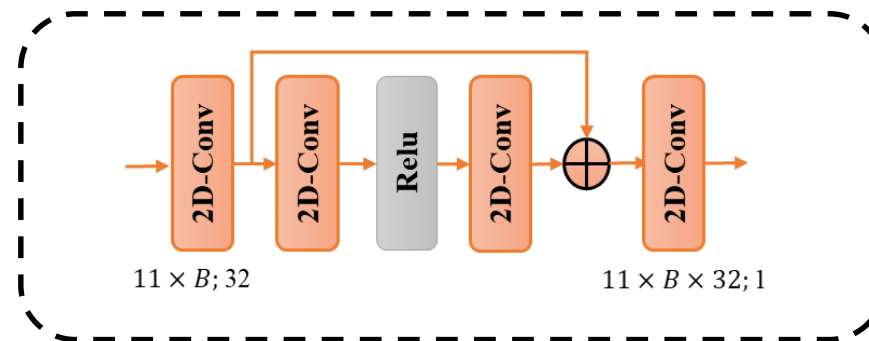
$$\mathbf{x}^k = \text{Concat}\left\{F^H \frac{(\bar{\mathbf{r}}_j^{k-1})^* \cdot \bar{\mathbf{s}}_j + 2\xi^k \bar{\mathbf{w}}_j^{k-1}}{(\bar{\mathbf{r}}_j^{k-1})^* \cdot \bar{\mathbf{r}}_j^{k-1} + 2\xi^k}\right\}$$

$$\mathbf{w}^k = \mathbf{w}^{k-1} - v^k (\mu^k \nabla \Phi(\mathbf{w}^{k-1}) + 2\xi^k (\mathbf{w}^{k-1} - \mathbf{x}^k))$$

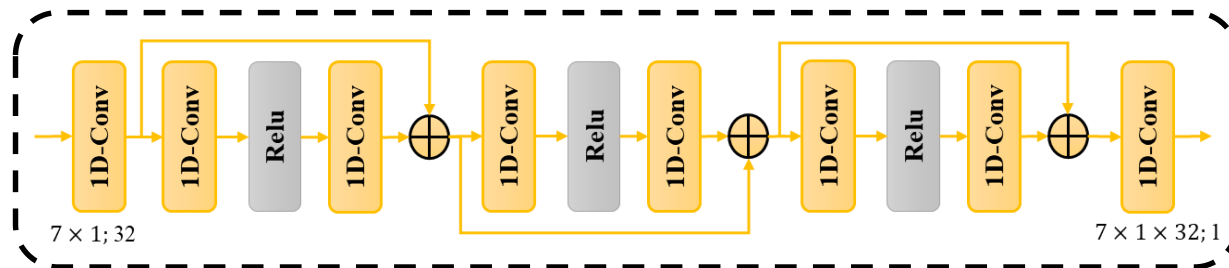
$$\mathbf{z}^k = \sum_{j=1}^B F^H \frac{(\bar{\mathbf{w}}_j^k)^* \cdot \bar{\mathbf{s}}_j + 2\beta^k \bar{\mathbf{r}}_j^{k-1}}{(\bar{\mathbf{w}}_j^k)^* \cdot \bar{\mathbf{w}}_j^k + 2\beta^k}$$

$$\mathbf{r}^k = \mathbf{r}^{k-1} - \zeta^k (\lambda^k \nabla \Psi(\mathbf{r}^{k-1}) + 2\beta^k (\mathbf{r}^{k-1} - \mathbf{z}^k))$$

$2DCNNs_{\Theta_1^k}$



$1DCNNs_{\Theta_2^k}$

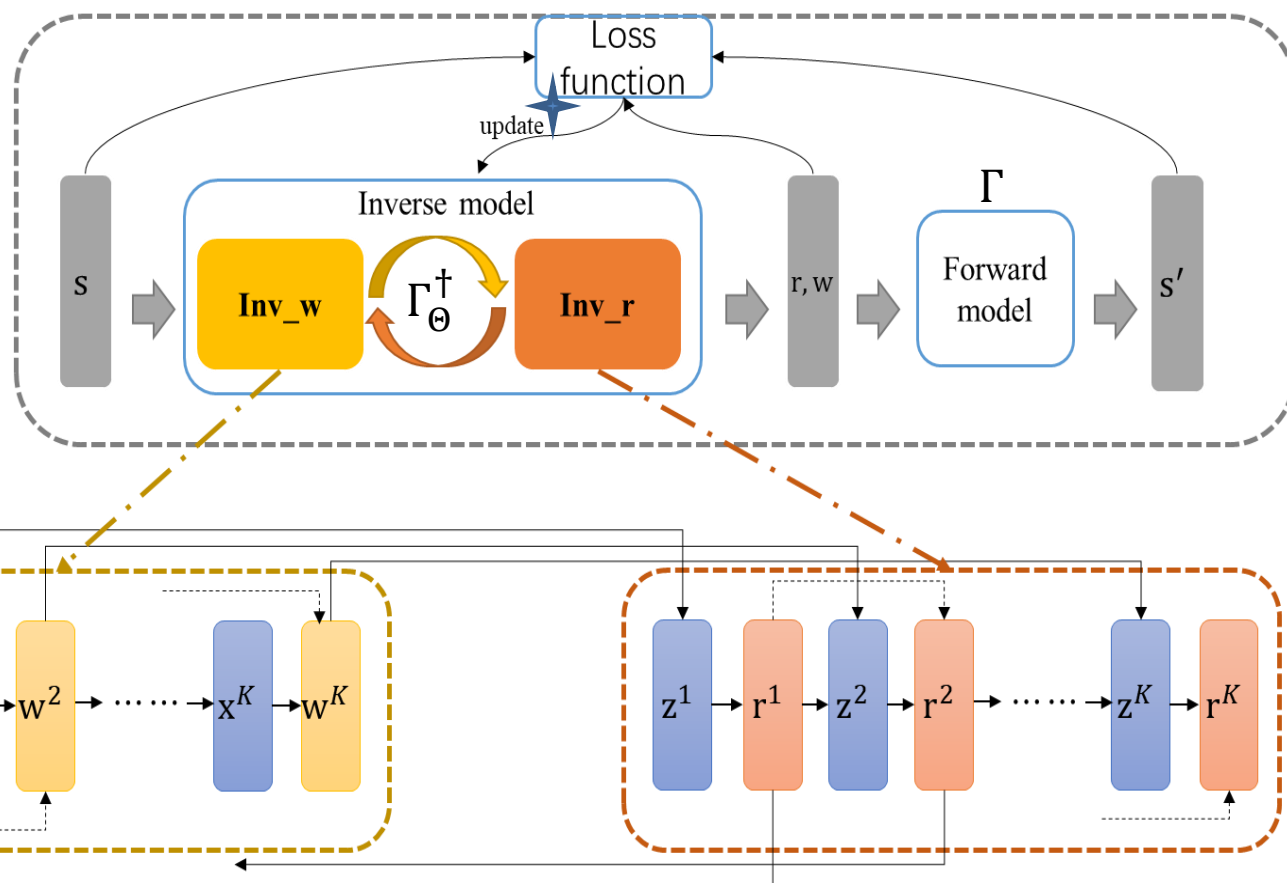


Theory

PMDDLMM: Deep learning based nonstationary SRI

Unrolling the iterations parts in equation (8):

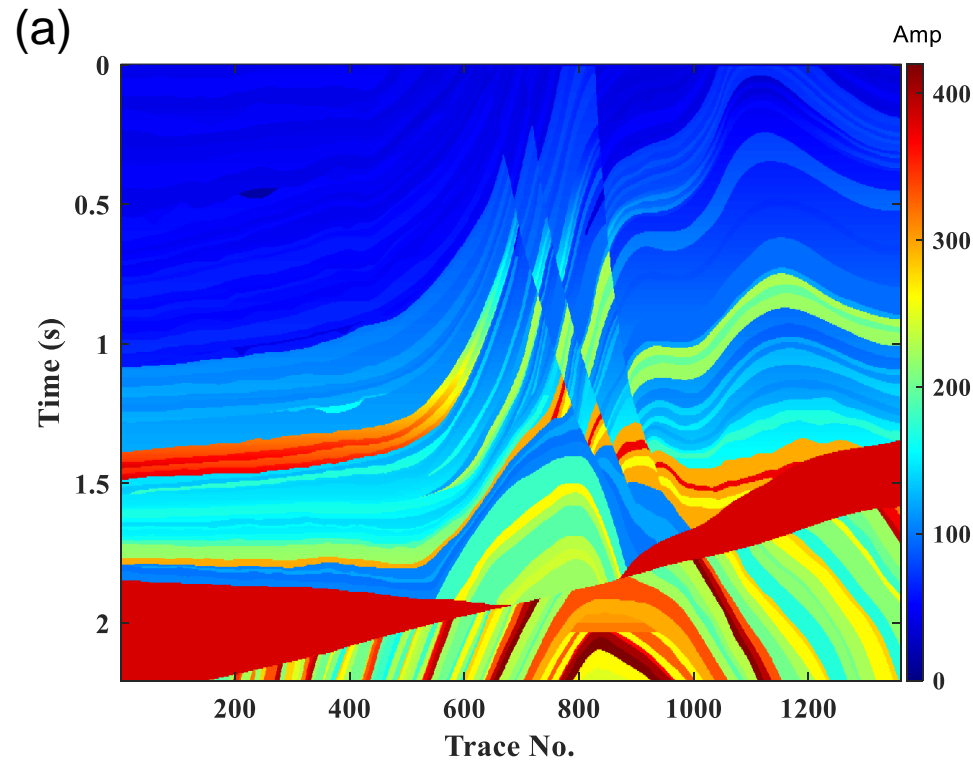
$$\begin{aligned}
 & E(\theta) \\
 &= \min_{\theta} \frac{1}{N \times N_t} \sum_{i=1}^N \|\Gamma_{\theta}^{\dagger}(\mathbf{s}_i)_{\mathbf{r}_i} - \mathbf{r}_i^{gt}\|_1 \quad \text{Supervised loss} \\
 &+ \frac{1}{N \times N_t} \sum_{i=1}^N \|\Gamma(\mathbf{r}_i^{gt}, \Gamma_{\theta}^{\dagger}(\mathbf{s}_i)_{\mathbf{w}_i}) - \mathbf{s}_i\|_1 \quad \text{Supervised loss} \\
 &+ \frac{1}{M \times N_t} \sum_{j=1}^M \|\mathbf{s}'_j - \mathbf{s}_j\|_1 \quad \text{Data-consistency loss} \\
 &+ \gamma \frac{1}{M \times B \times M_t} \sum_{j=1}^M \|D\mathbf{w}\|_1 \quad \text{smooth loss}
 \end{aligned}
 \tag{9}$$



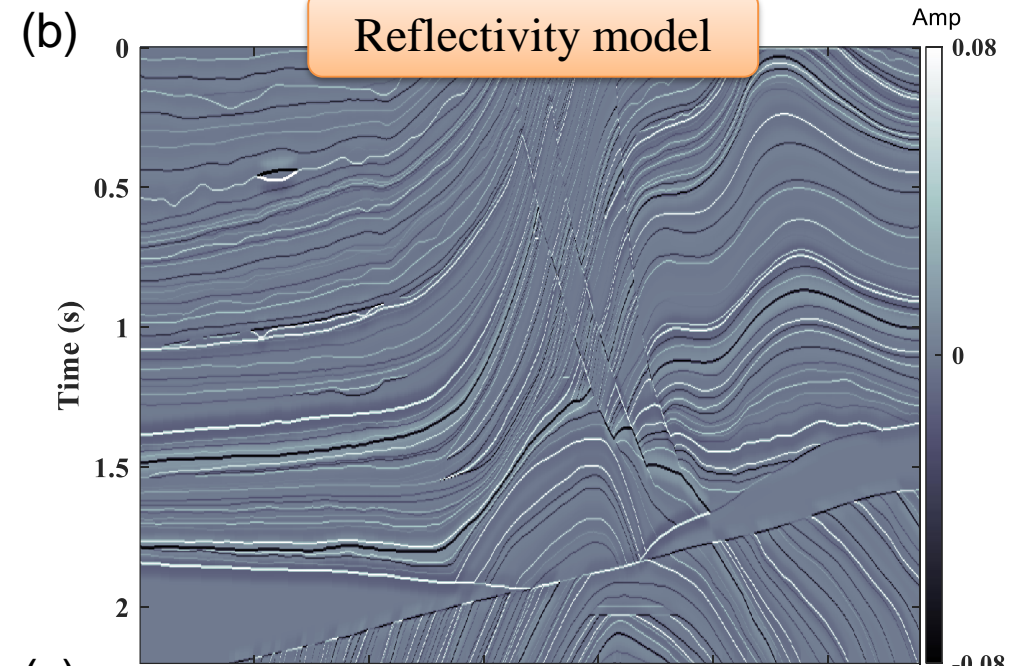
Examples

Examples

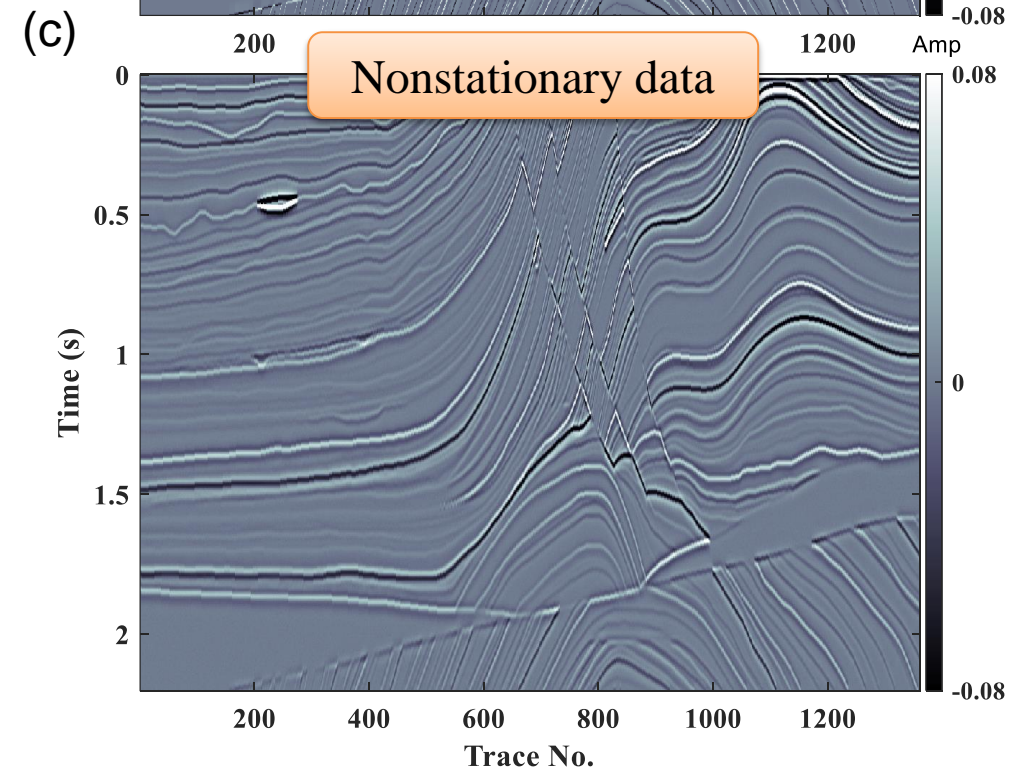
□ Synthetic data example



Interval Q model



Reflectivity model



Nonstationary data

Examples

□ Synthetic data example

$$\text{PCC} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

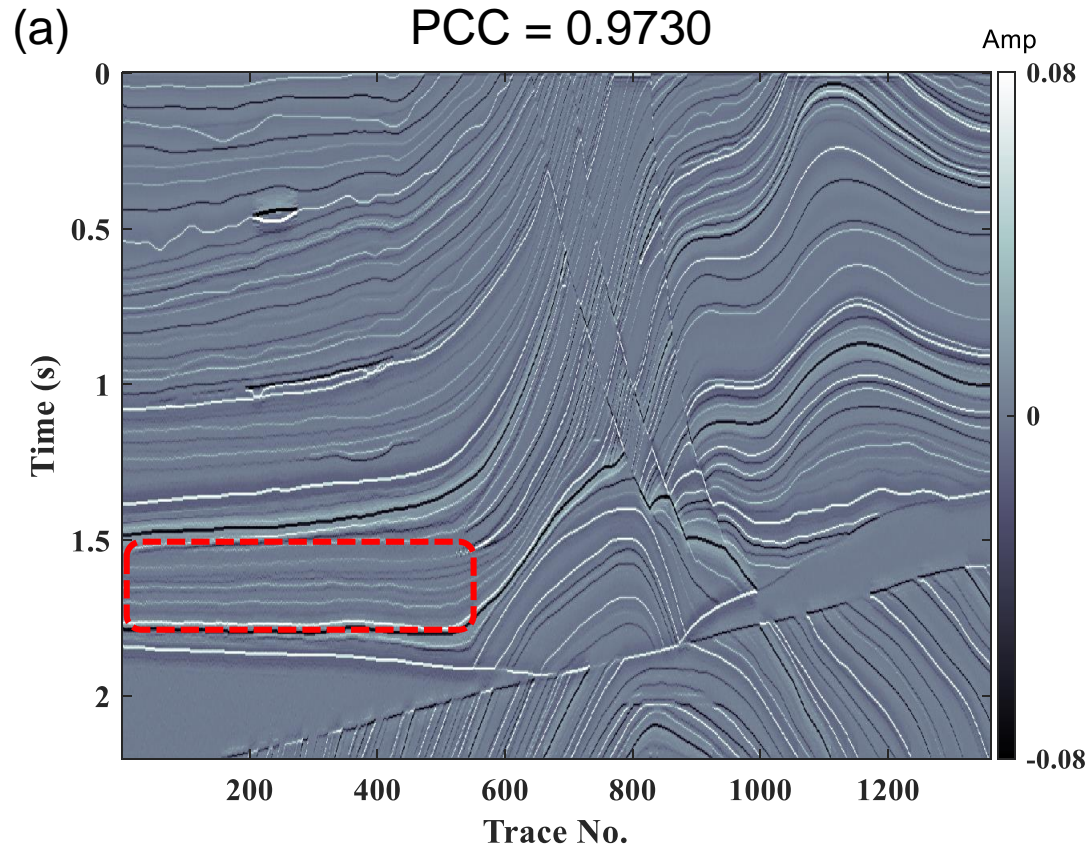
r = correlation coefficient

x_i = values of the x-variable in a sample

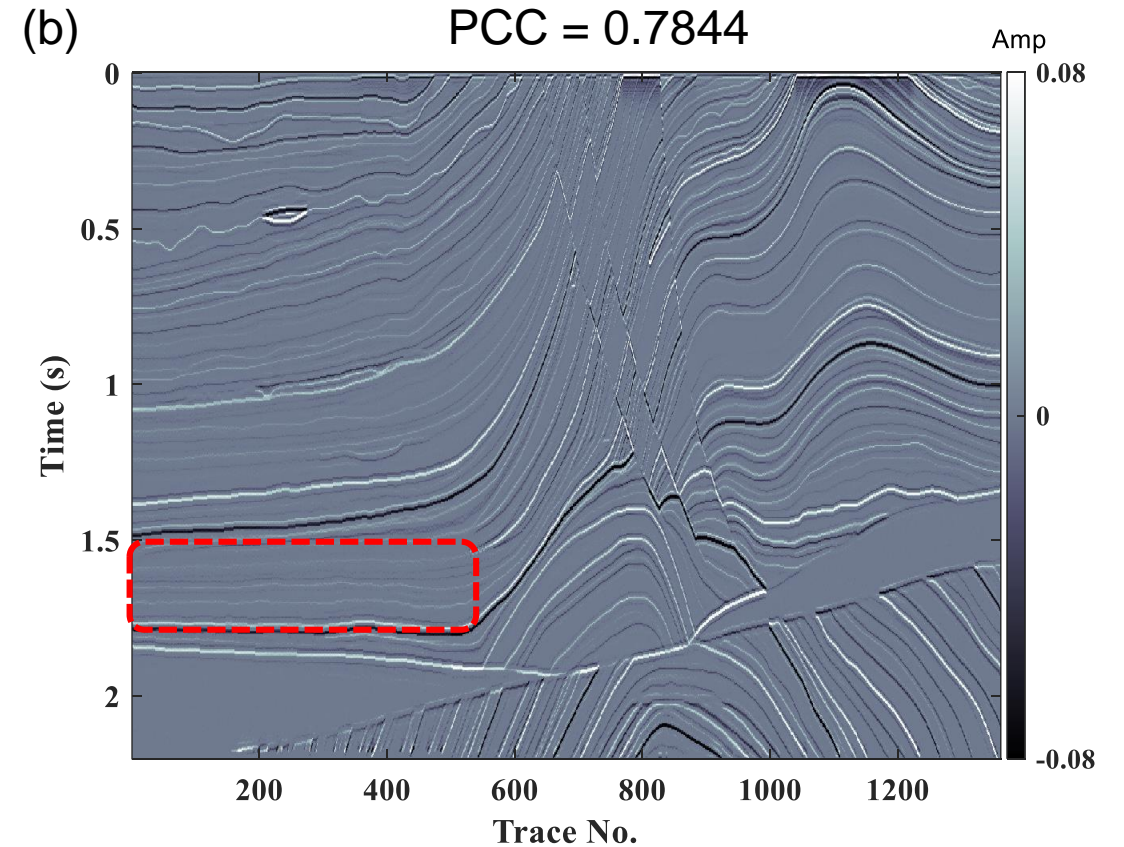
\bar{x} = mean of the values of the x-variable

y_i = values of the y-variable in a sample

\bar{y} = mean of the values of the y-variable



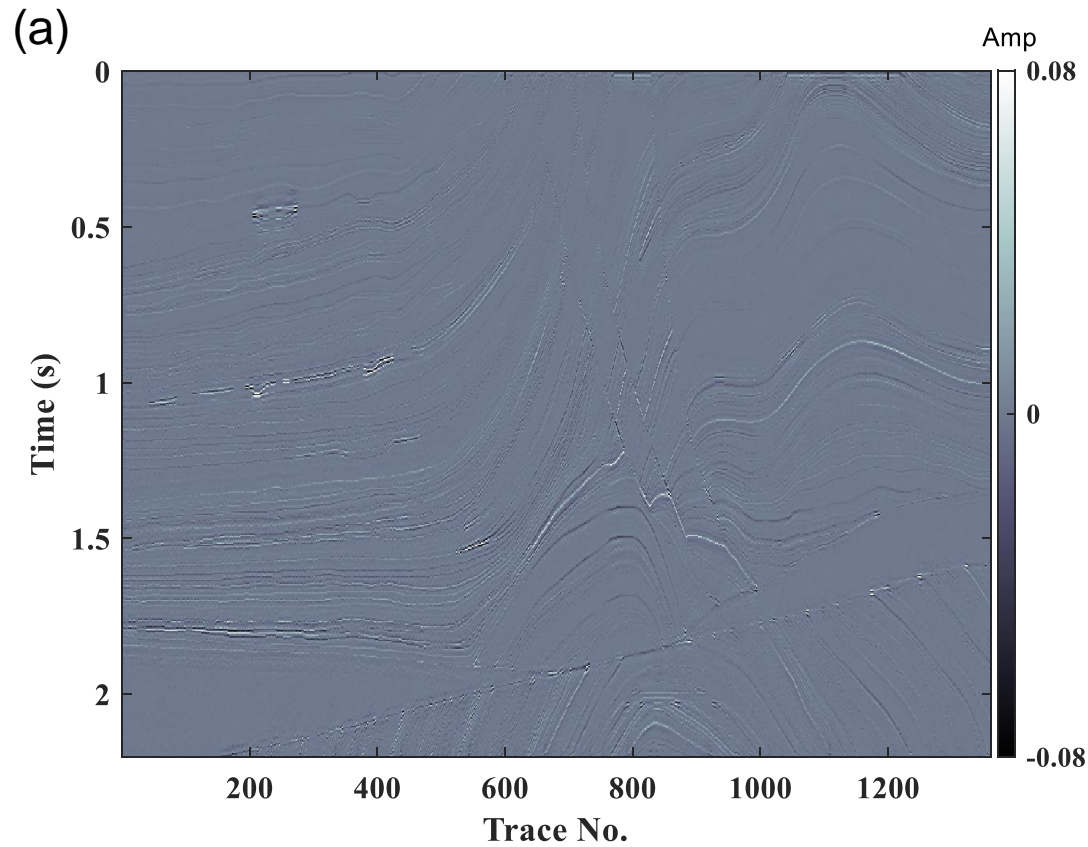
DL inverted reflectivity



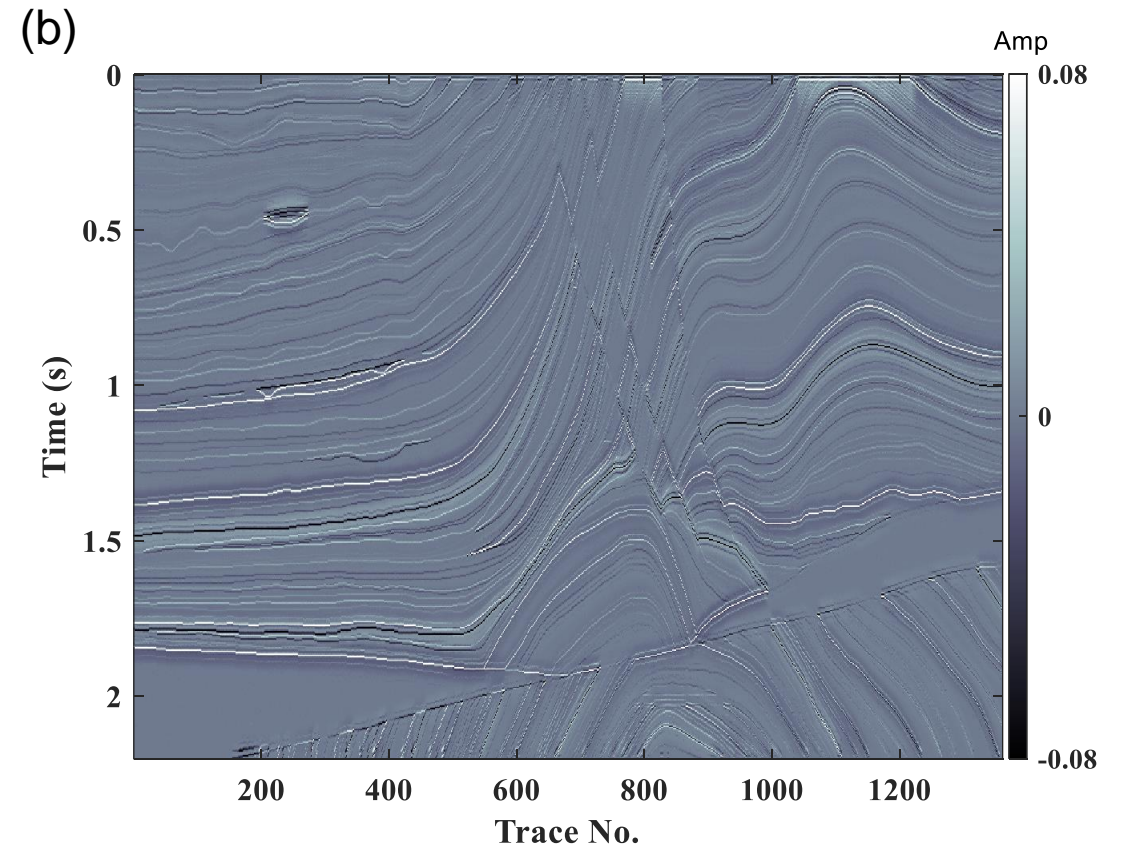
NBD inverted reflectivity

Examples

□ Synthetic data example



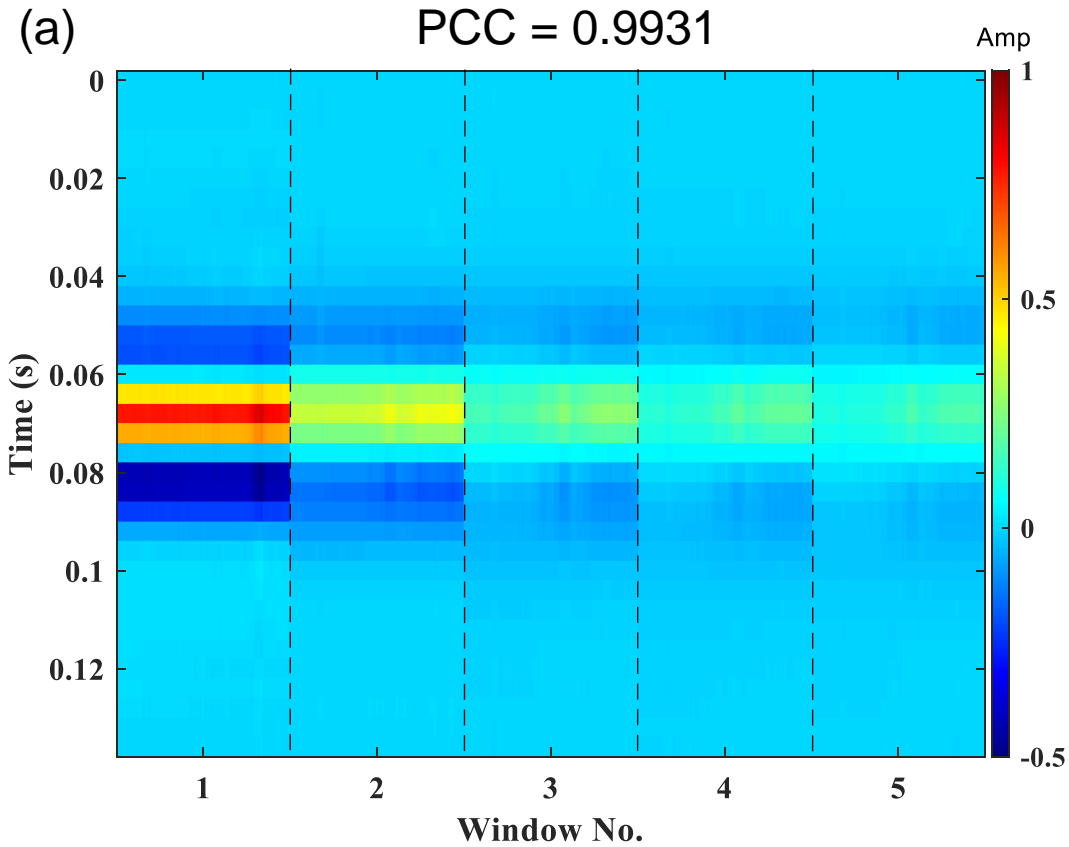
DL inverted reflectivity error



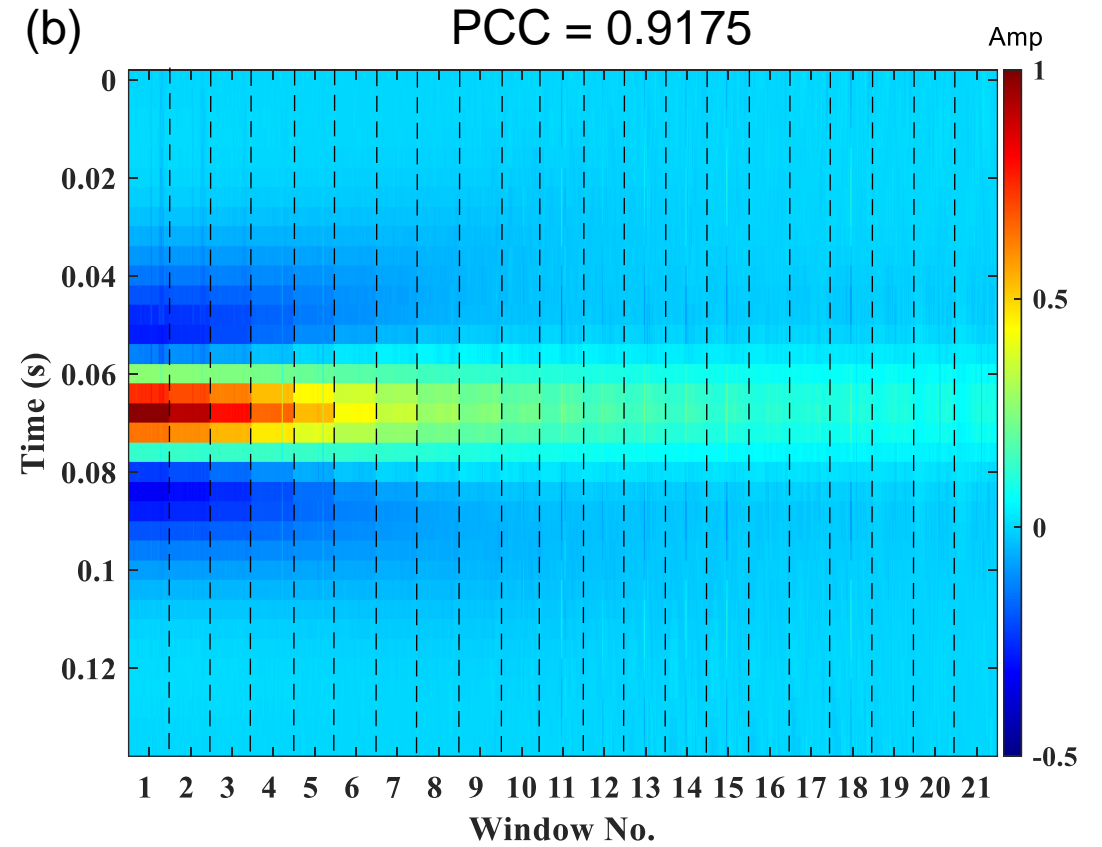
NBD inverted reflectivity error

Examples

□ Synthetic data example



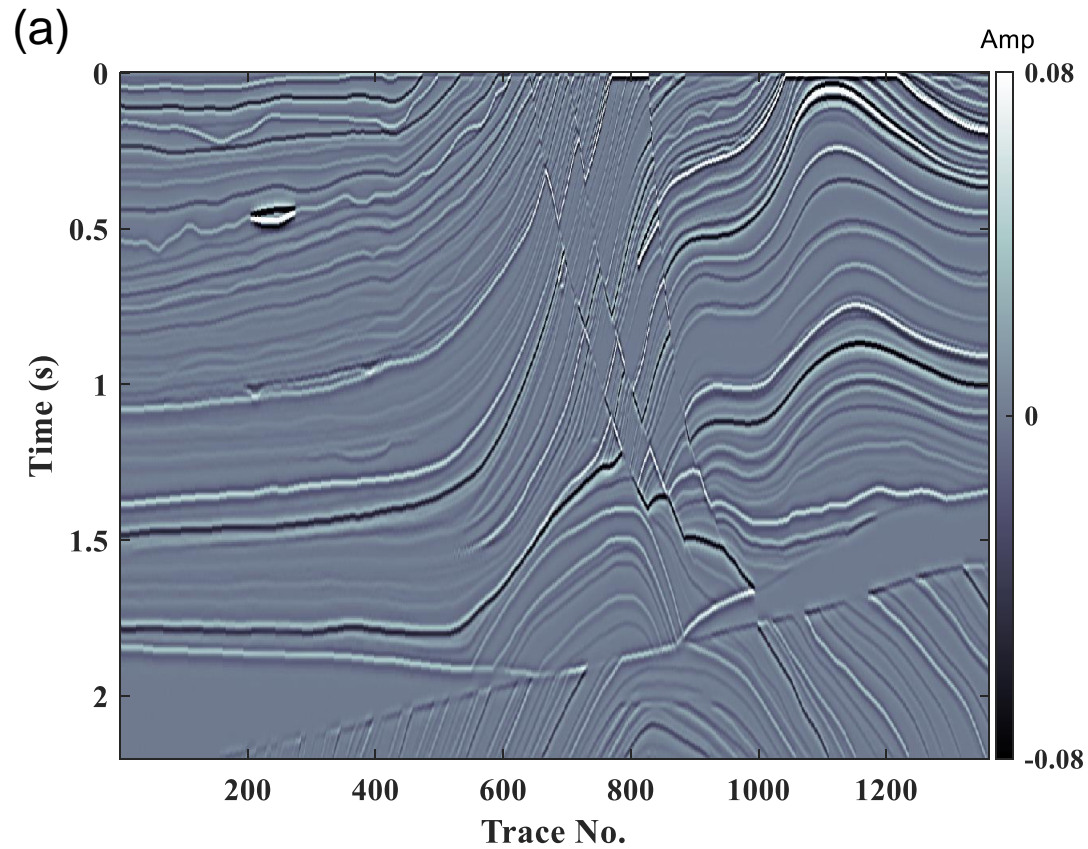
DL inverted wavelets



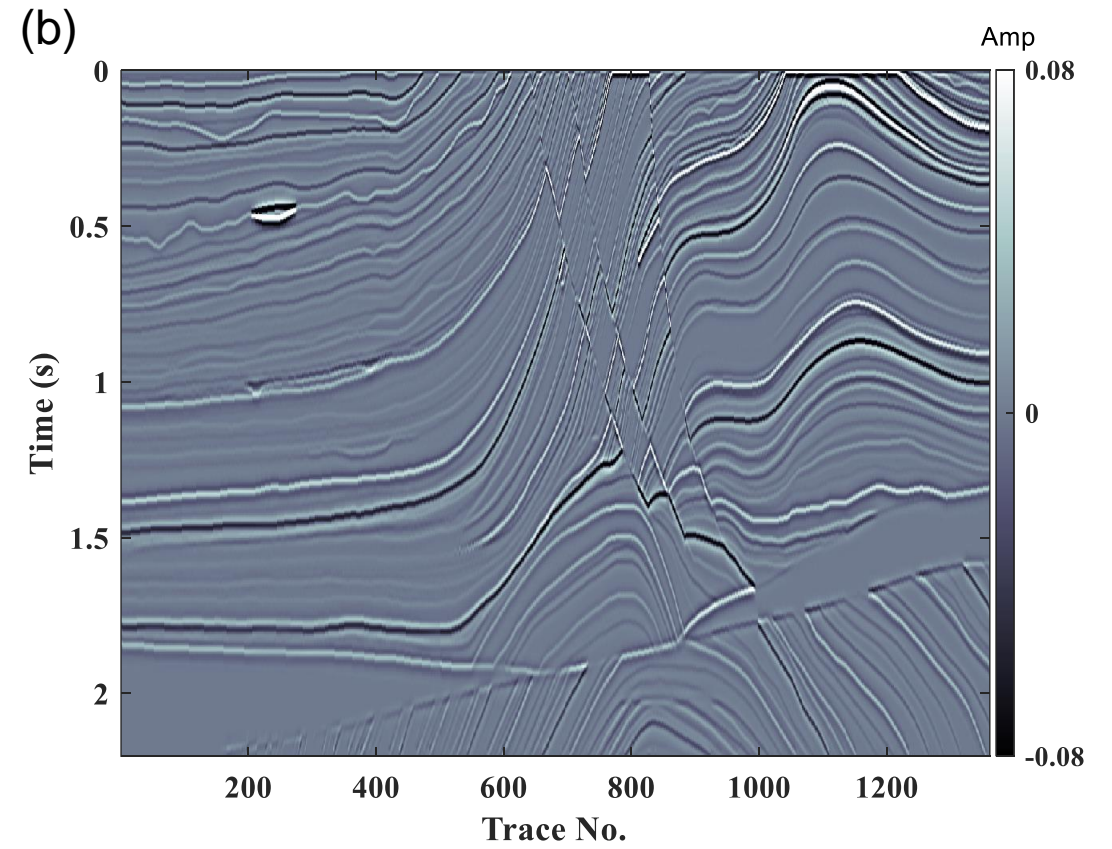
NBD inverted wavelets

Examples

□ Synthetic data example



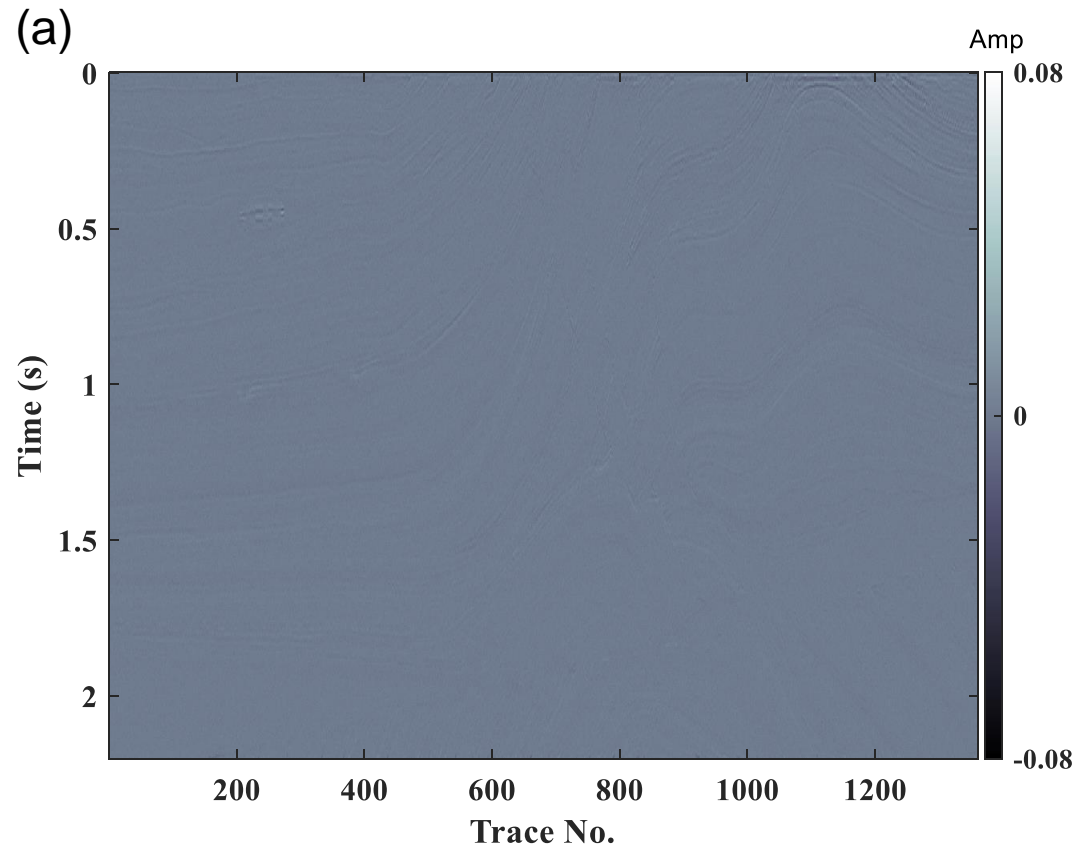
DL reconstructed data



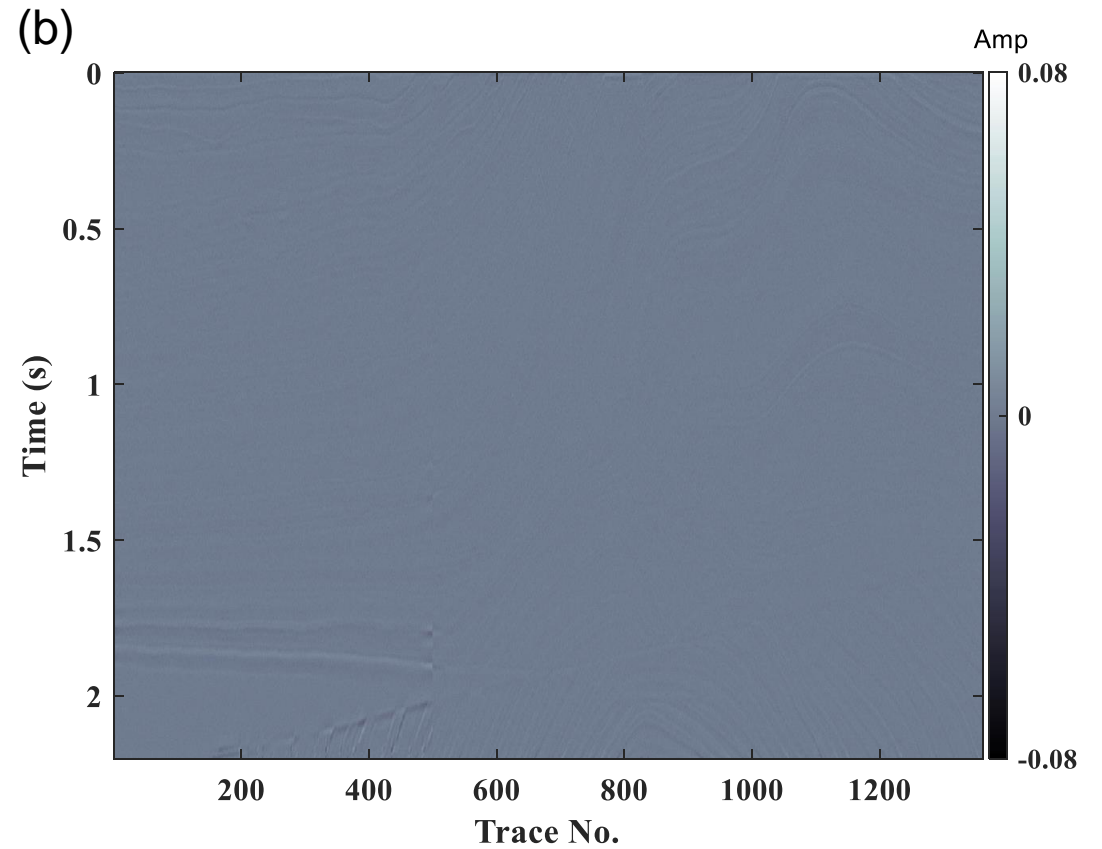
NBD reconstructed data

Examples

□ Synthetic data example



DL reconstructed data error

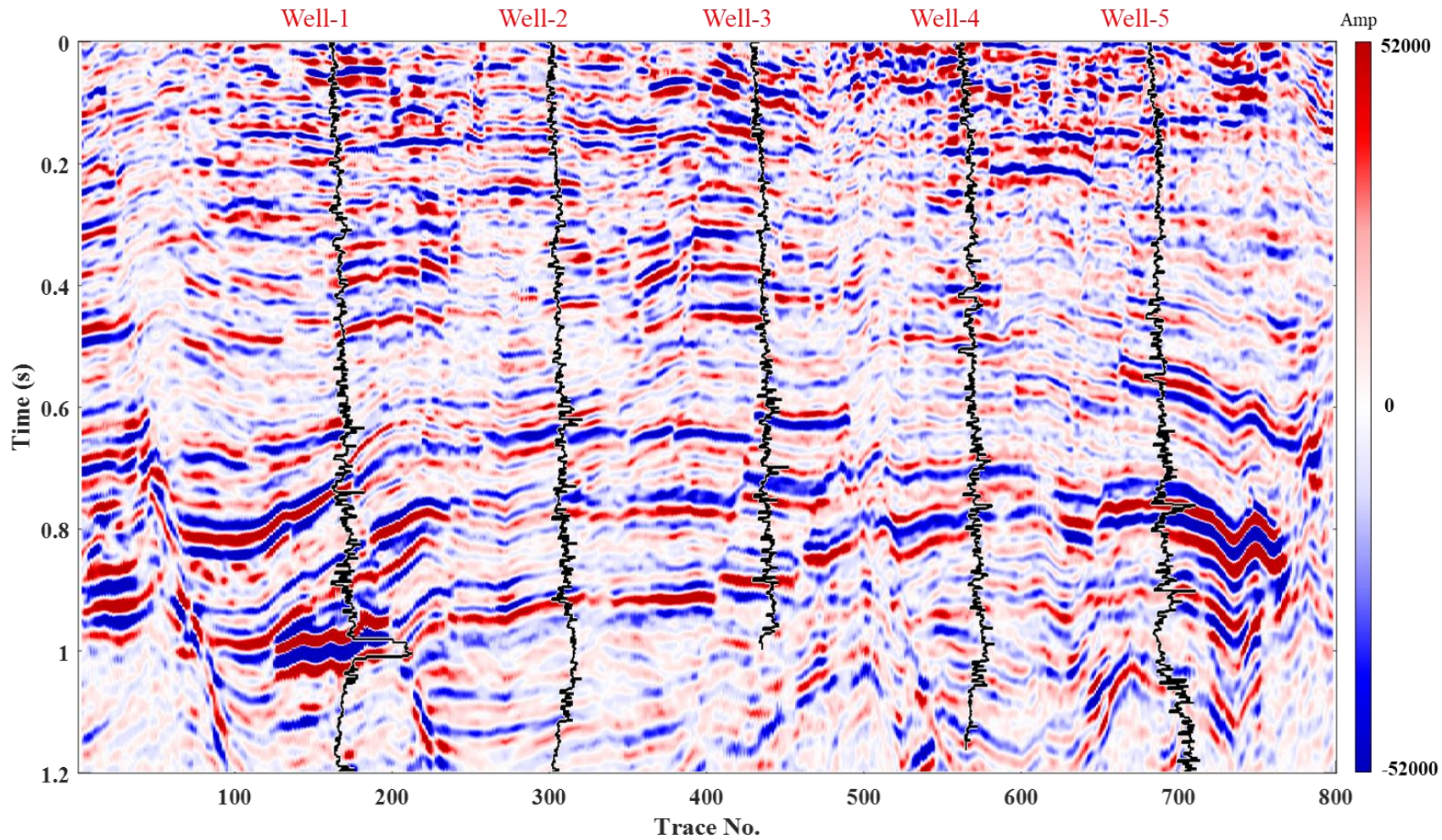


NBD reconstructed data error

Examples

Field data example

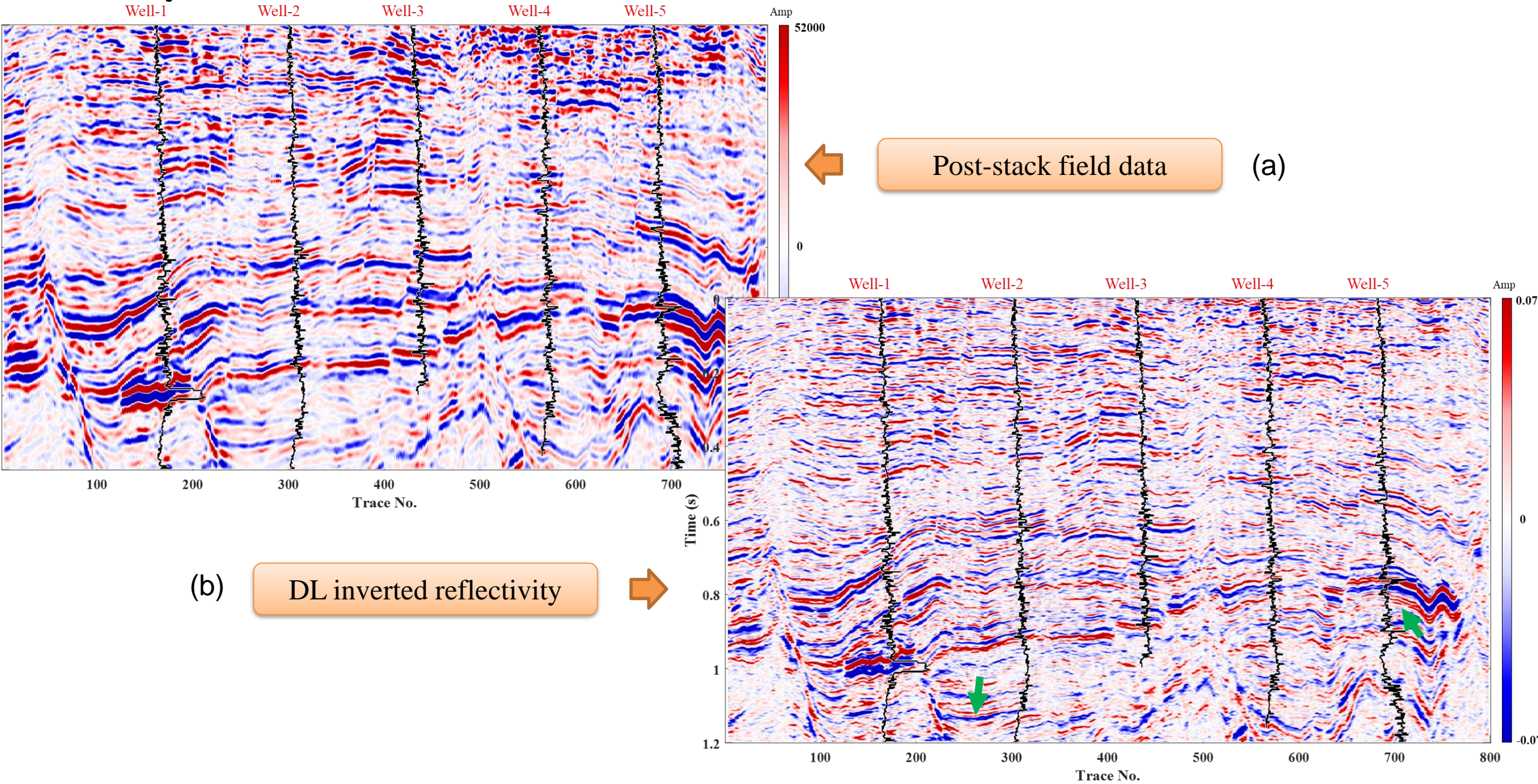
Post-stack field data



- Using well-1, well-2, well-3, well-5 to train and well-4 to validate.

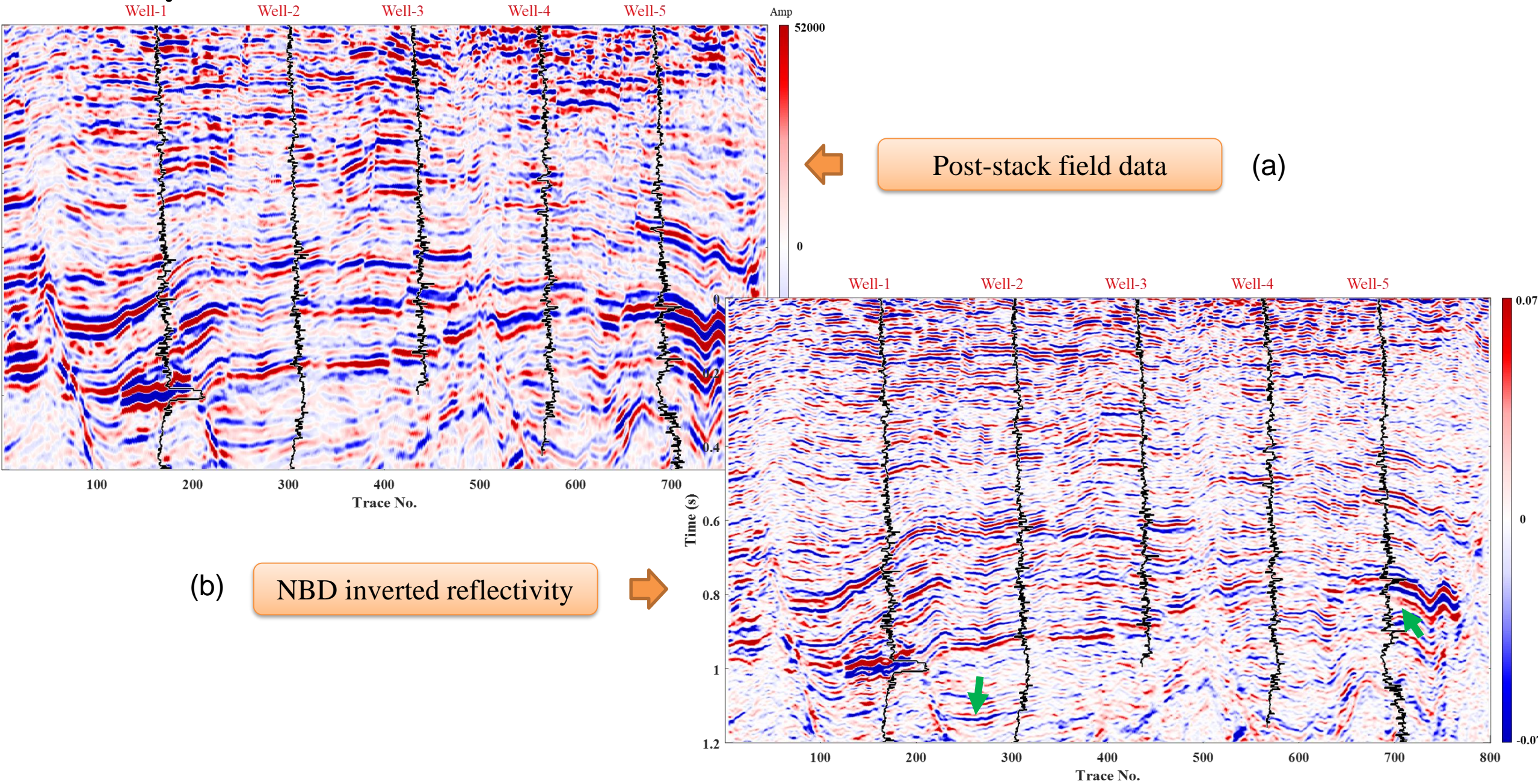
Examples

□ Field data example



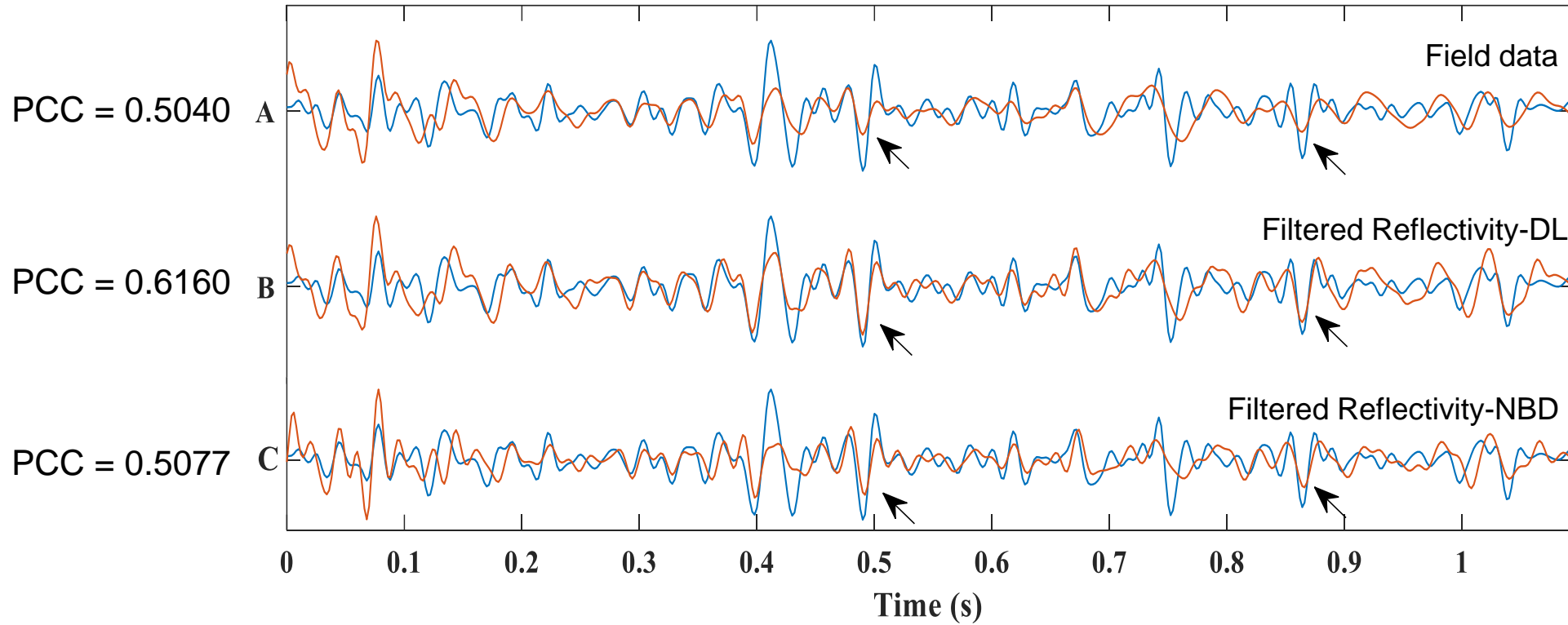
Examples

□ Field data example



Examples

Field data example

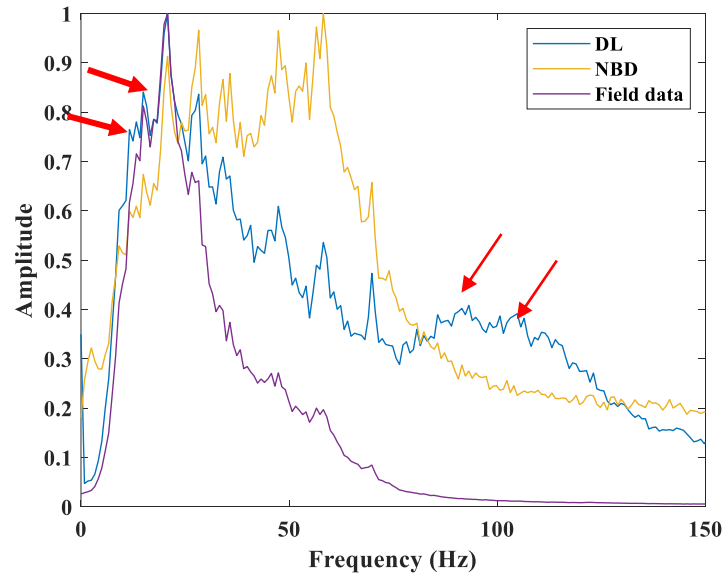


filtered reflectivity of well-4

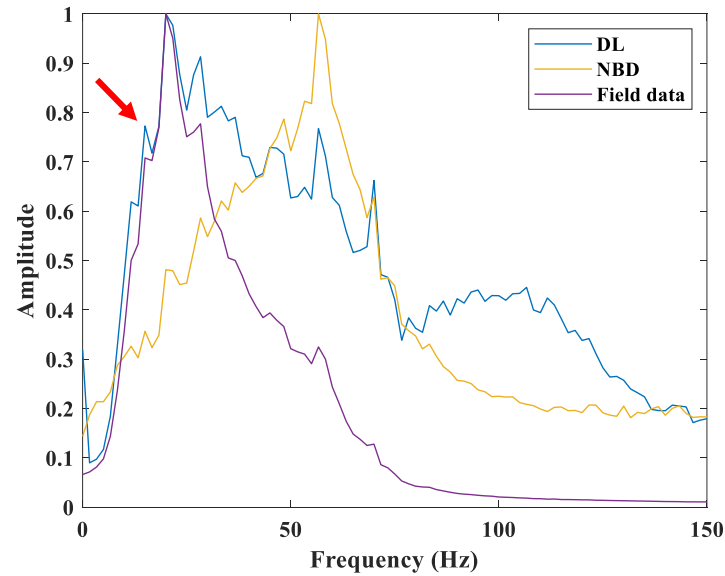
Examples

□ Field data example

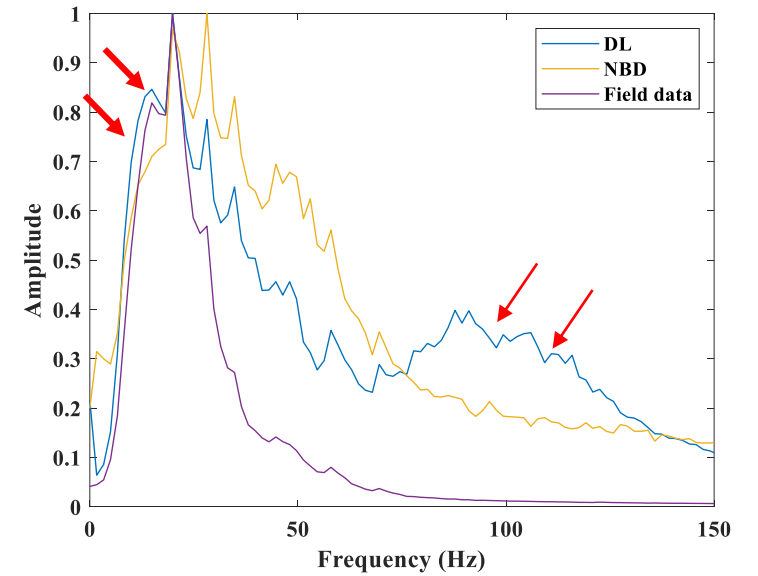
(a)



(b)



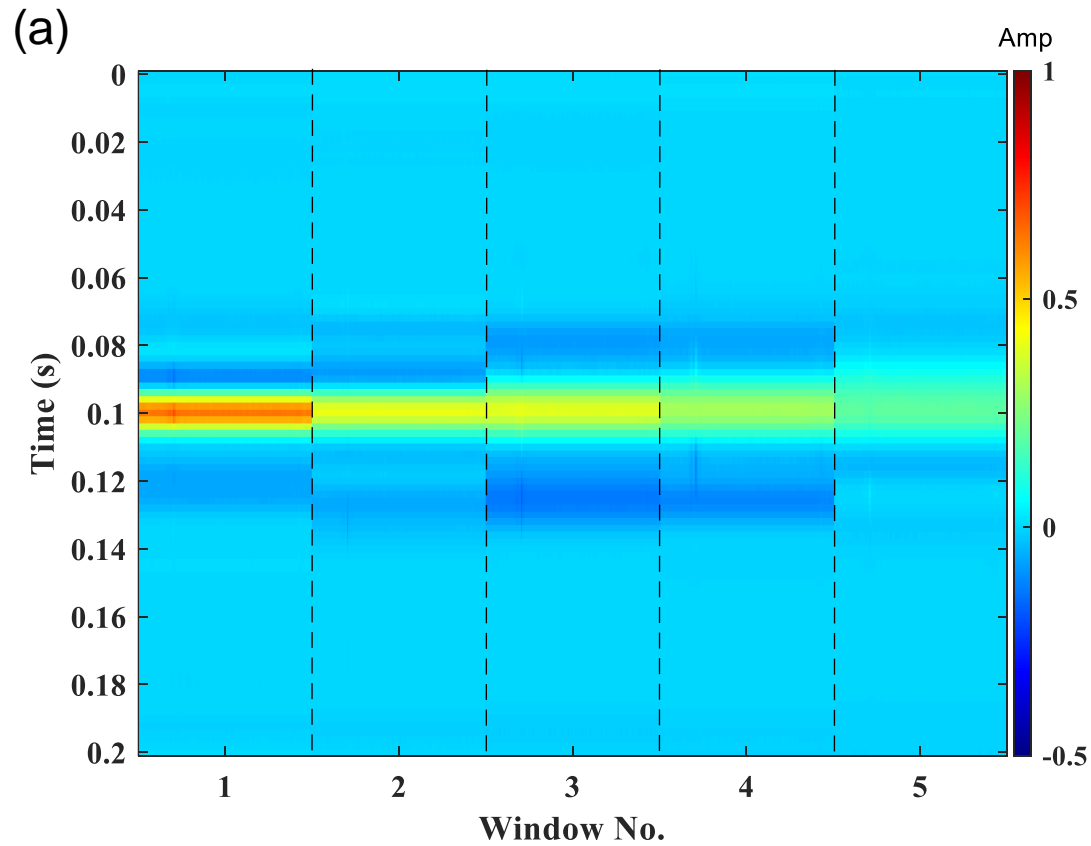
(c)



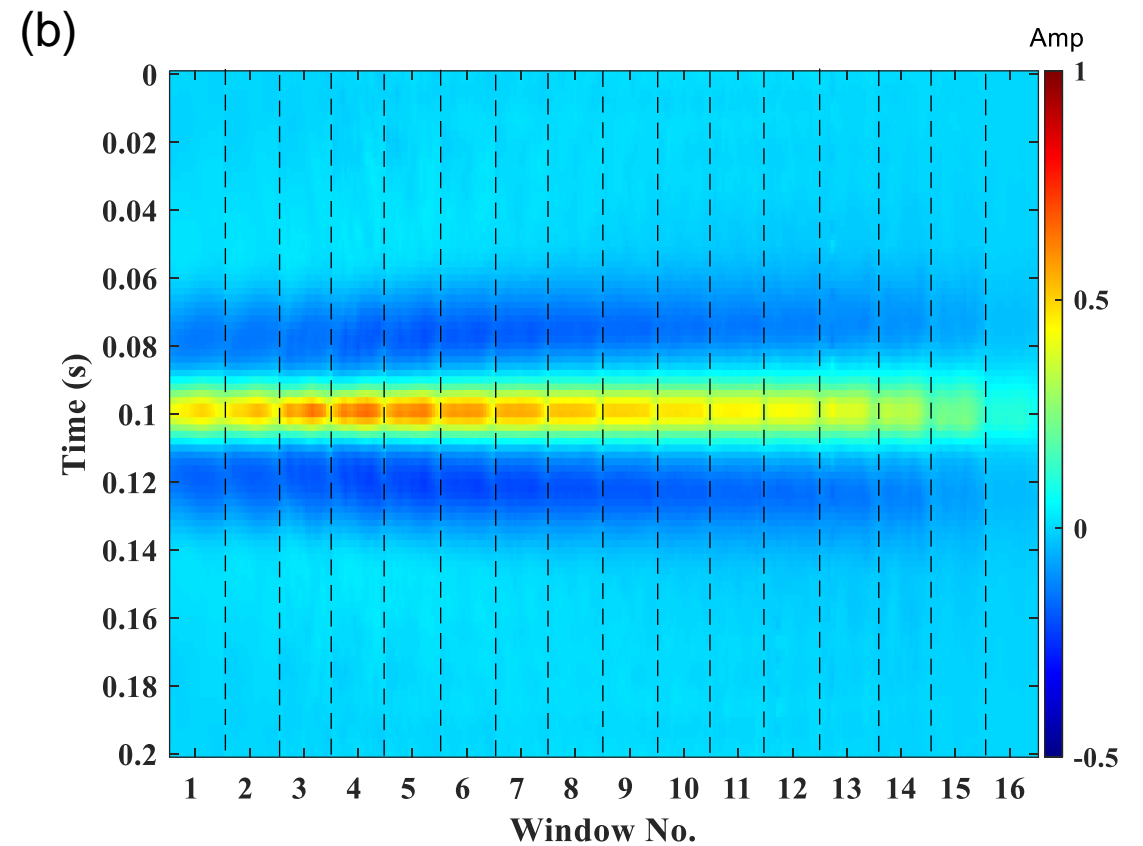
Amplitude spectra of the entire data profile, the first half part, and the second half part

Examples

Field data example



DL inverted wavelets



NBD inverted wavelets

Conclusions

- We build a prior-engaged neural network framework by unrolling an alternating iterative optimization algorithm to simultaneously estimate the reflectivity and time-varying wavelets;
- We introduce two data-consistency losses to learn the time-varying wavelets and transfer the knowledge from the unlabeled data;
- We add a regularization term in the loss function to constrain the time-varying wavelets to make them smooth in the spatial direction;
- Some experiments are conducted to show the effectiveness of the proposed method.

Acknowledgments

- Sponsors of the Signal Analysis and Imaging Group at the University of Alberta
- CNOOC providing the field data