ND Robust interpolation by orthogonal matching pursuit with Fourier Operators

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Outline

- Introduction
- Theory
- Examples
- Conclusions

Introduction

Seismic Interpolation

- Minimum weighted norm interpolation (MWNI)
- Antileakage Fourier transform (ALFT)
- Projection on convex sets (POCS)
- Matching Pursuit Fourier Interpolation (MPFI)

Theory

Traditional MP Fourier interpolation

Step 1: Transfer the data from t-x domain to f-x domain by FFT

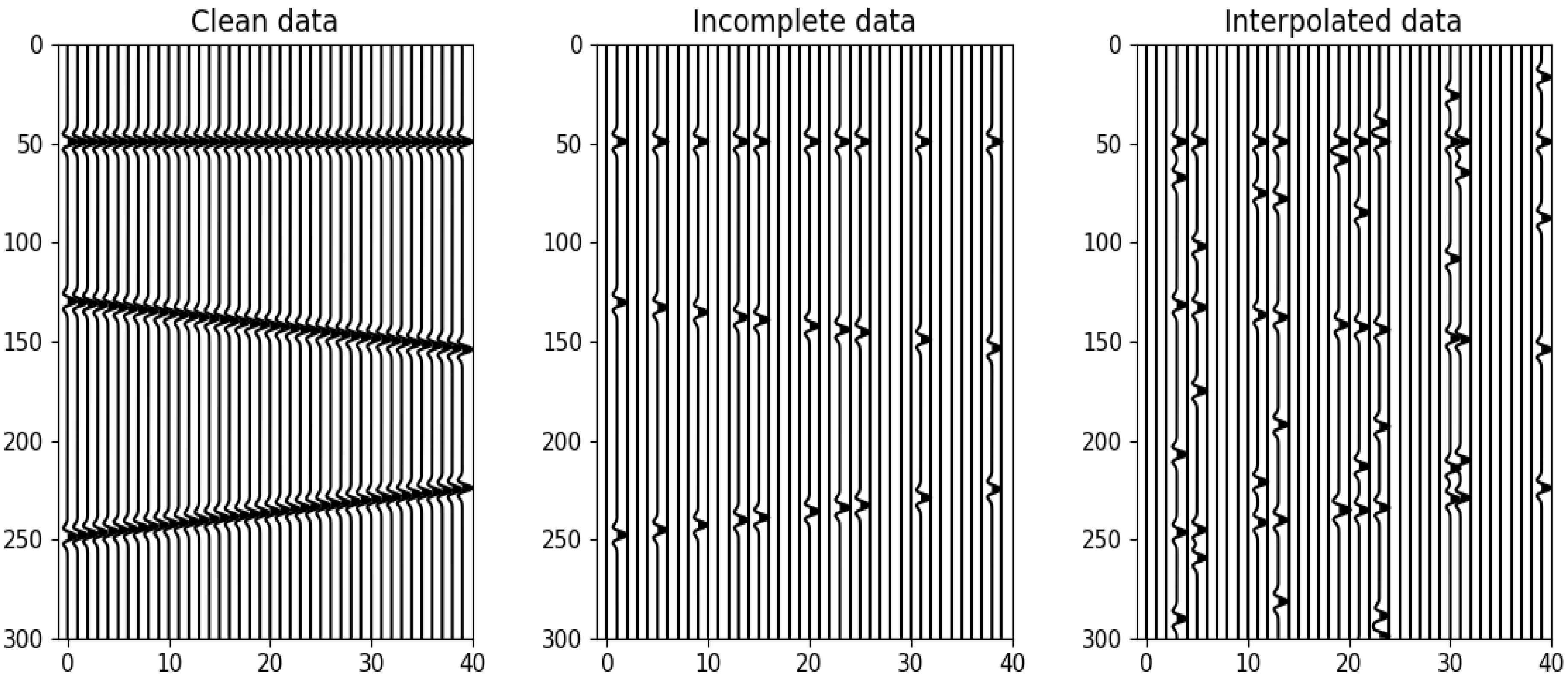
Step 2: For each frequency slice

- 2.1:Compute the Fourier domain using a discrete Fourier transform (DFT)
- 2.2:Find the Fourier component with maximum energy
- 2.3:Add this Fourier component to the estimated spectrum
- 2.4:Transform this Fourier component back to the input locations
- 2.5:Subtract the result from step 4 from the input data for this iteration
- 2.6:Repeat 2.1 to 2.5

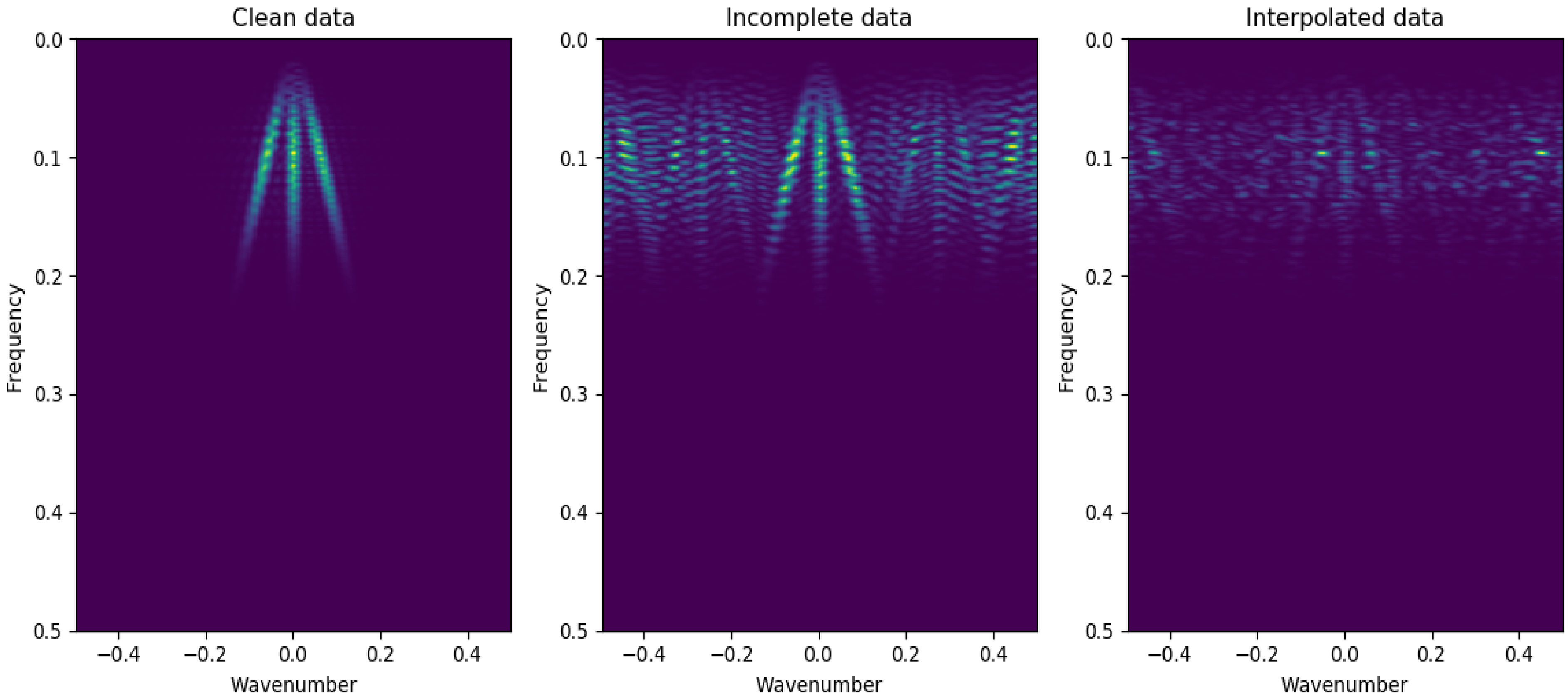
Step 3: Repeat step 2 for all frequency slices, transfer the result back to t-x domain by inverse FFT

Problem: results are sub-optimal due to aliases
can't separate the erratic noise in one frequency slice.

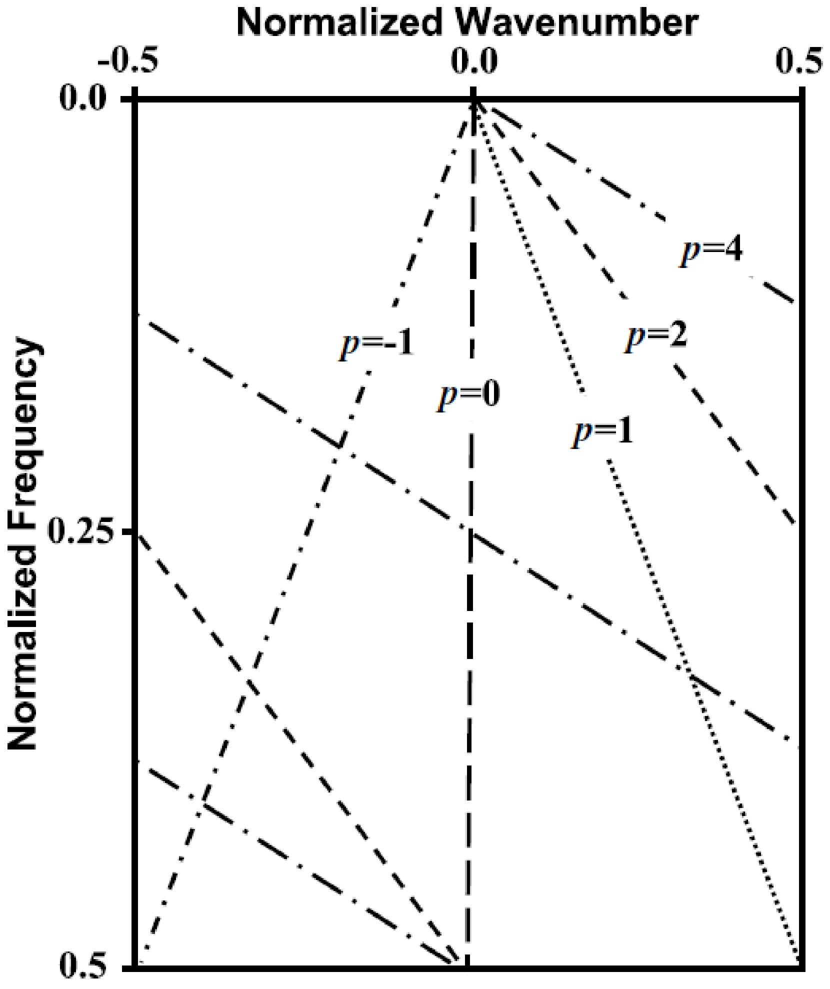
Synthetic example



F-K spectra

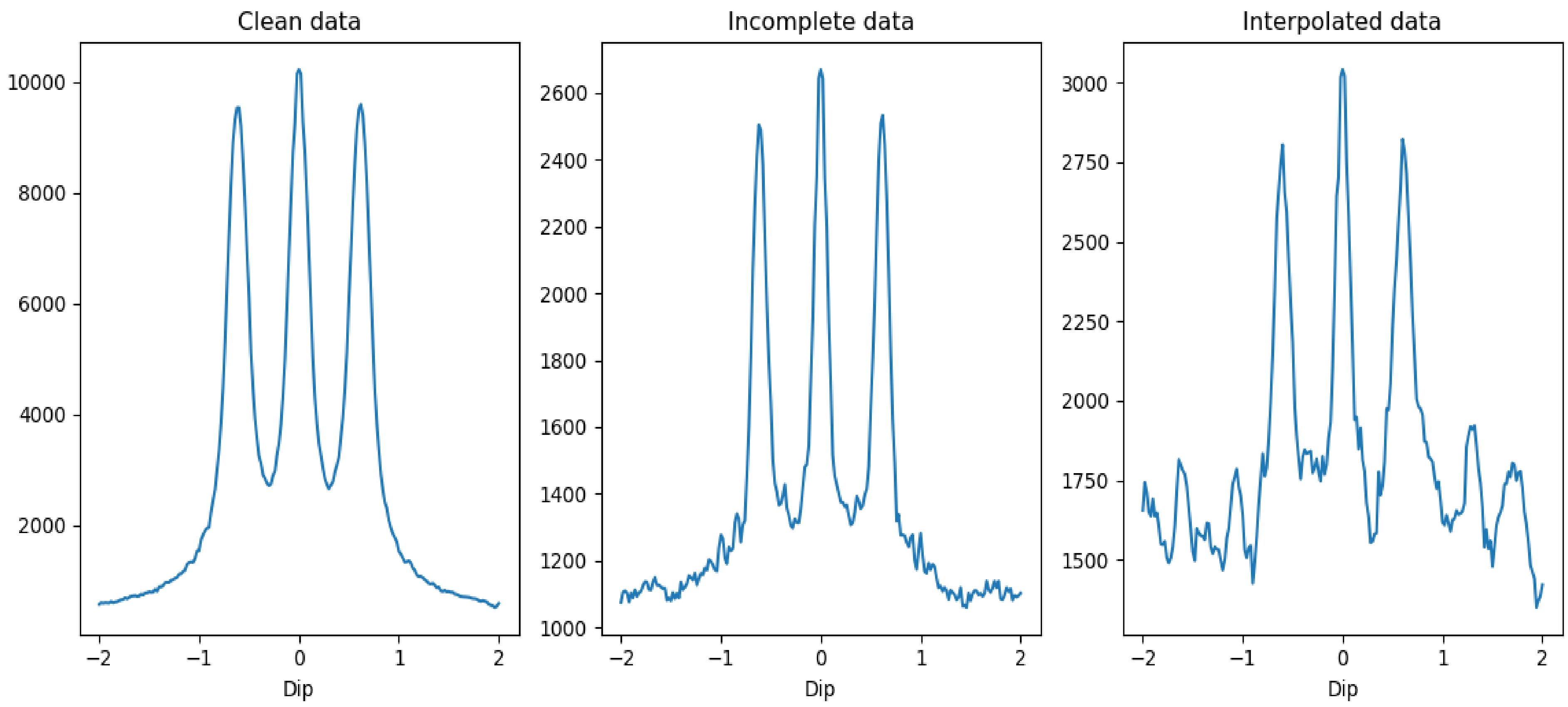

Clean data
Incomplete data
Interpolated data
Frequency
Wavenumber
0.0
0.1
0.2
0.3
0.4
0.5
−0.4
−0.2
0.0
0.2
0.4

Angular search



$$M(p) = \sum_{n=1}^{N_\omega} D(\omega_n, k = p\omega_n - \lfloor p\omega_n + 0.5 \rfloor),$$

Angular search



Greedy Algorithm

Matching Pursuit (MP)

Orthogonal Matching Pursuit (OMP)

Two basic steps
1: Find the best correlated basis function

Find the dominant dip with maximum energy.

2: Update the basis function to fit the data

M-estimator

M-estimator

- L1
- L1-L2
- Huber
- Cauchy
- Tukey

Robust OMP Fourier interpolation

Input: $\mathbf{dn}$, and max iteration K

Output: $\mathbf{dr}$

Initialization: $\mathbf{r}^{[0]} = \mathbf{dn}$,$\widehat{\mathbf{x}}^{[k]} = 0$, and $T^{[0]} = \emptyset$

for $k = 1, 2, ..., K$ **do**

$m = fft2(r)$

Finding the event with dip p which has the maximum summation of the total energy in the $f-k$ domain.

Picking all coefficients along the dip p,$\widehat{\mathbf{x}}^{[k]}$

$\widehat{\mathbf{x}}^{[k]}_{T^{[k]}} = \text{argmin}_{\widetilde{\mathbf{x}}_{T^{[k]}}} \|\mathbf{y} - ifft2(\widetilde{\mathbf{x}}_{T^{[k]}})\|_\rho$

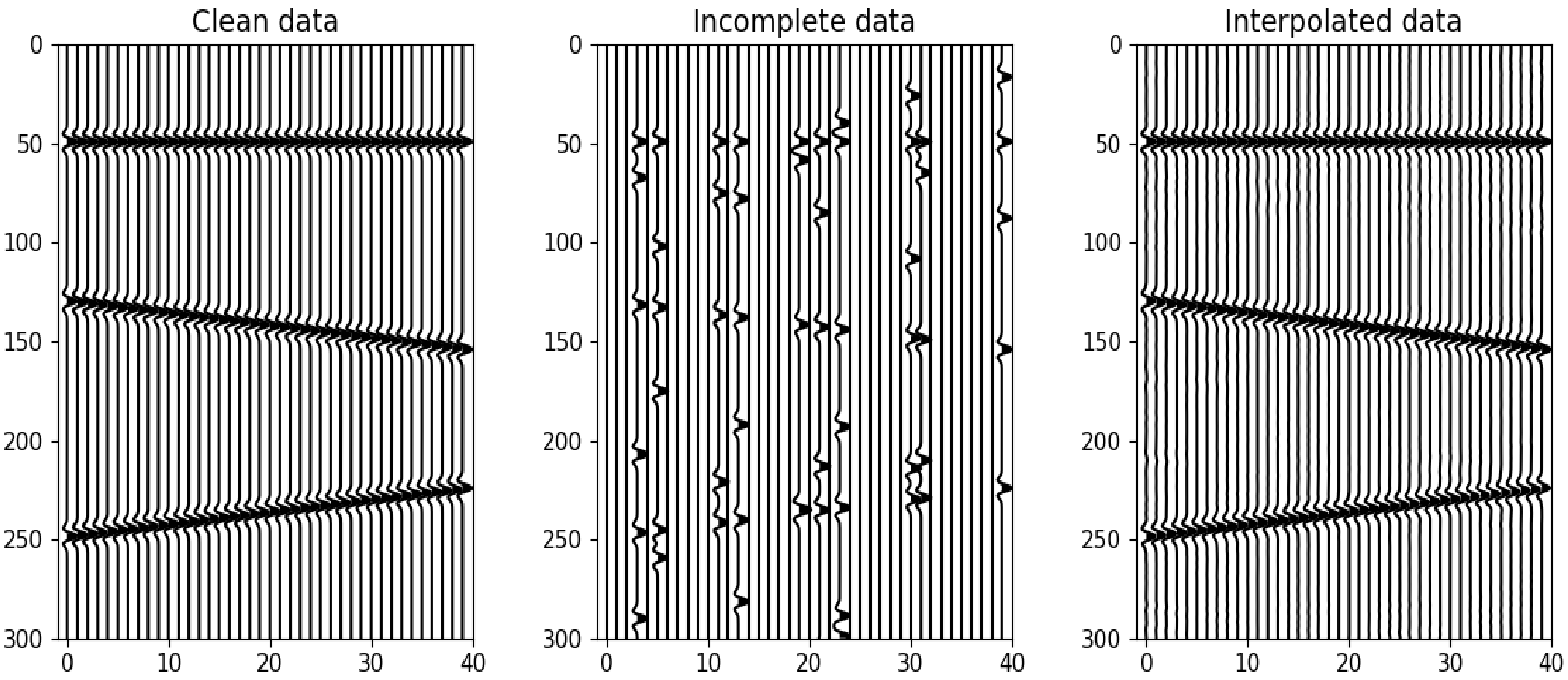
$\mathbf{r}^{[k]} = \mathbf{r} - ifft2(\widehat{\mathbf{x}}^{[k]}_{T^{[k]}})$

$\mathbf{dr}^{[k]} = \mathbf{dr} + ifft2(\widehat{\mathbf{x}}^{[k]}_{T^{[k]}})$

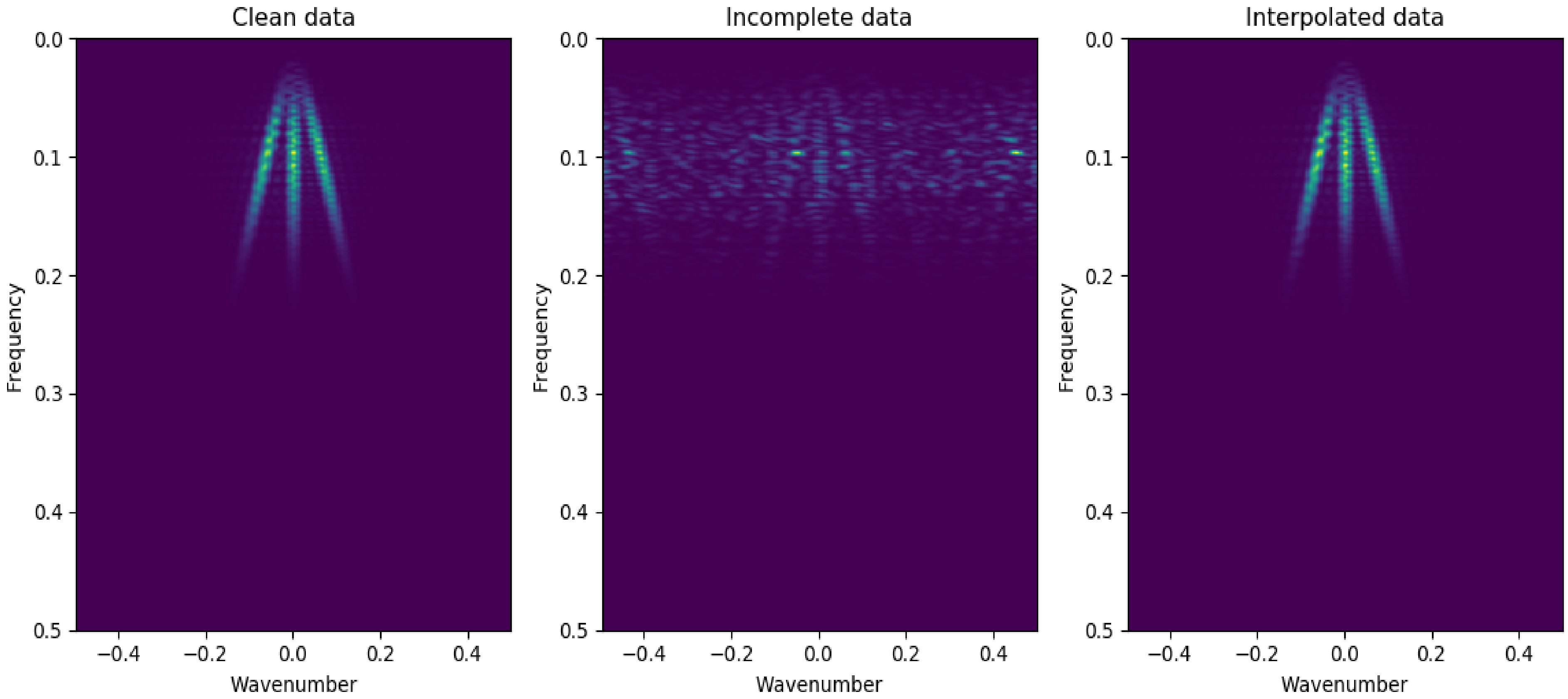
end for

Examples

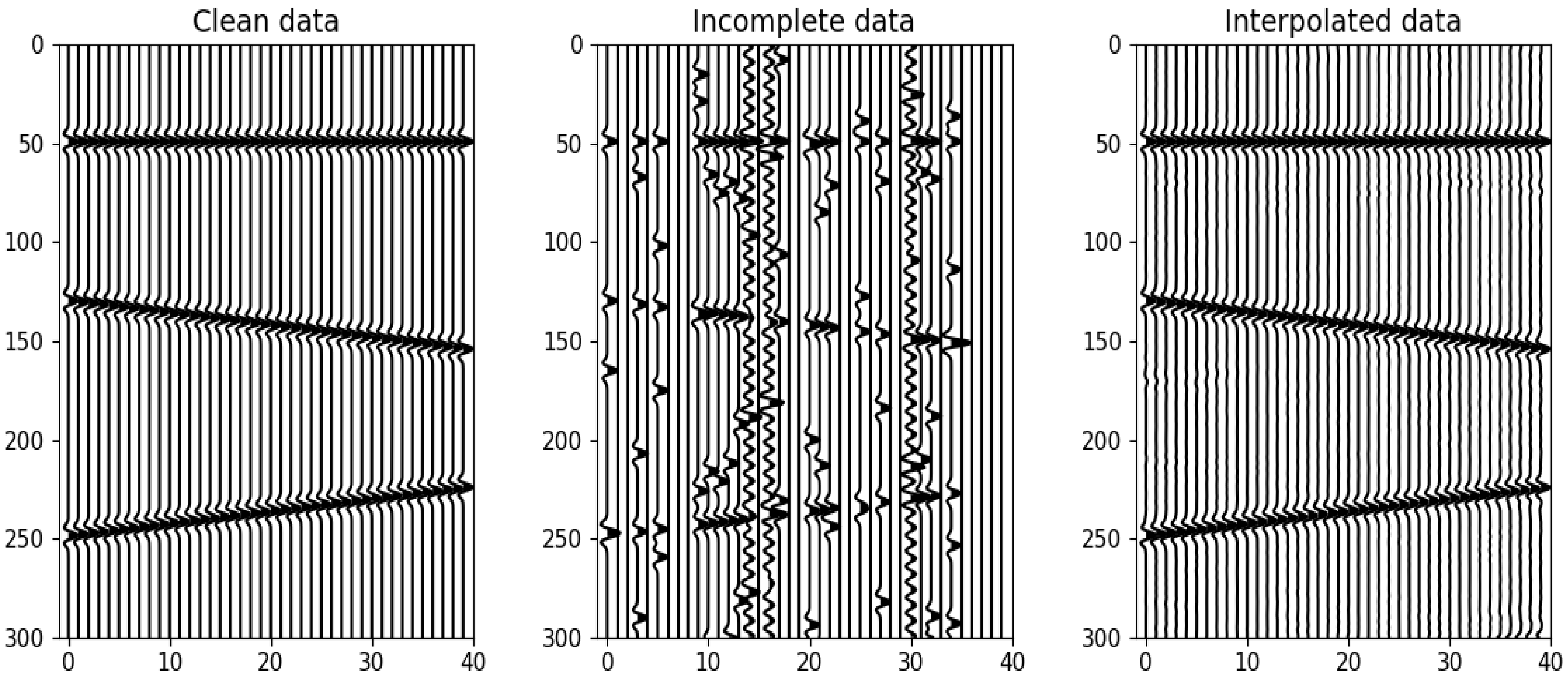
2D Synthetic example



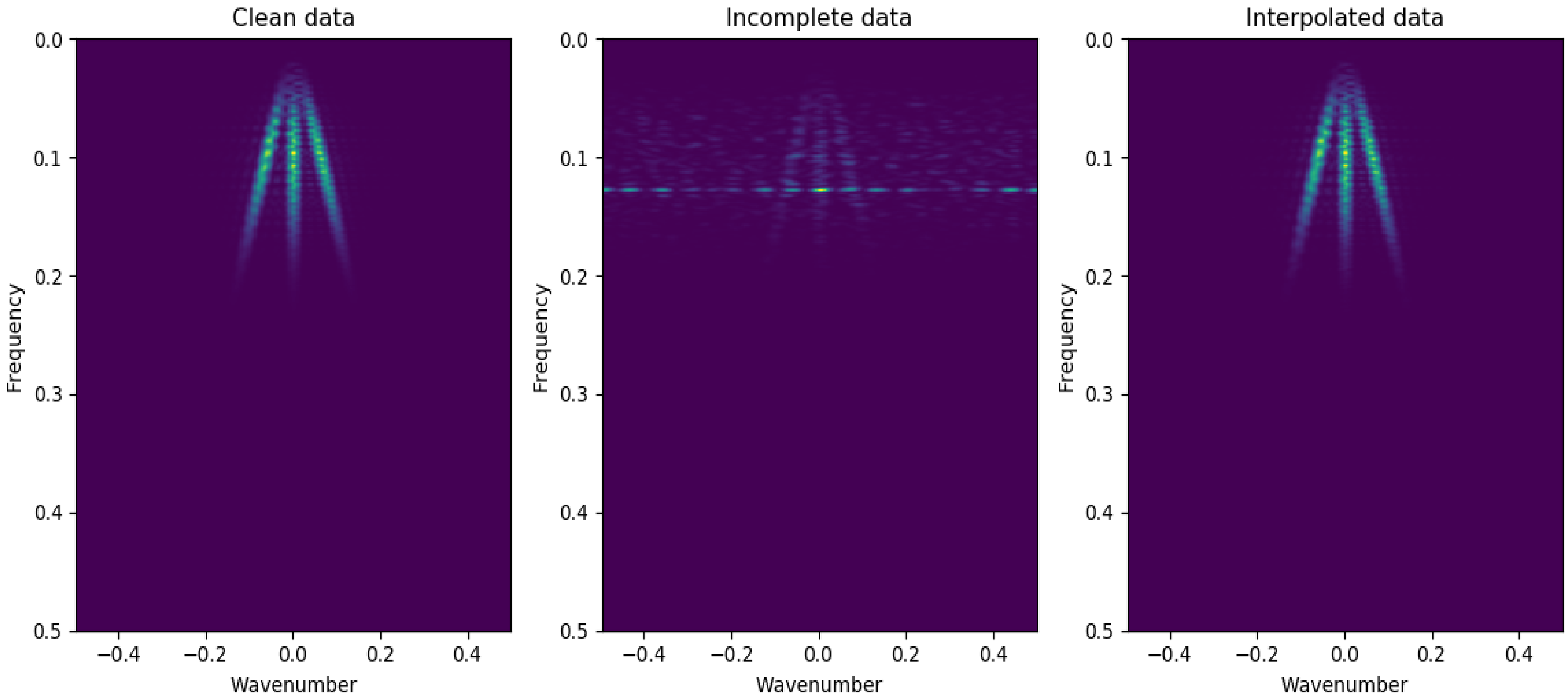
F-K spectra


Clean data
Incomplete data
Interpolated data
Frequency
Wavenumber
0.0
0.1
0.2
0.3
0.4
0.5
−0.4
−0.2
0.0
0.2
0.4

2D Synthetic example

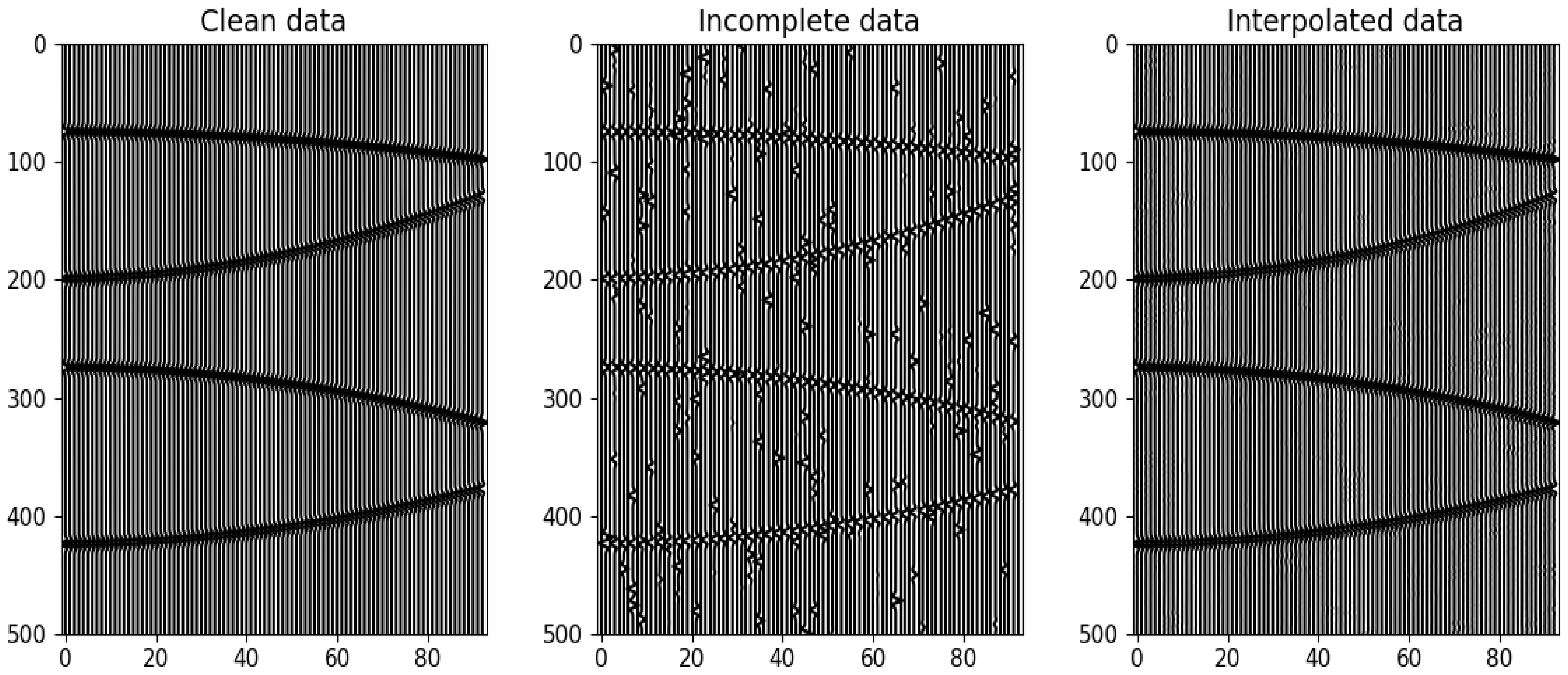


F-K spectra

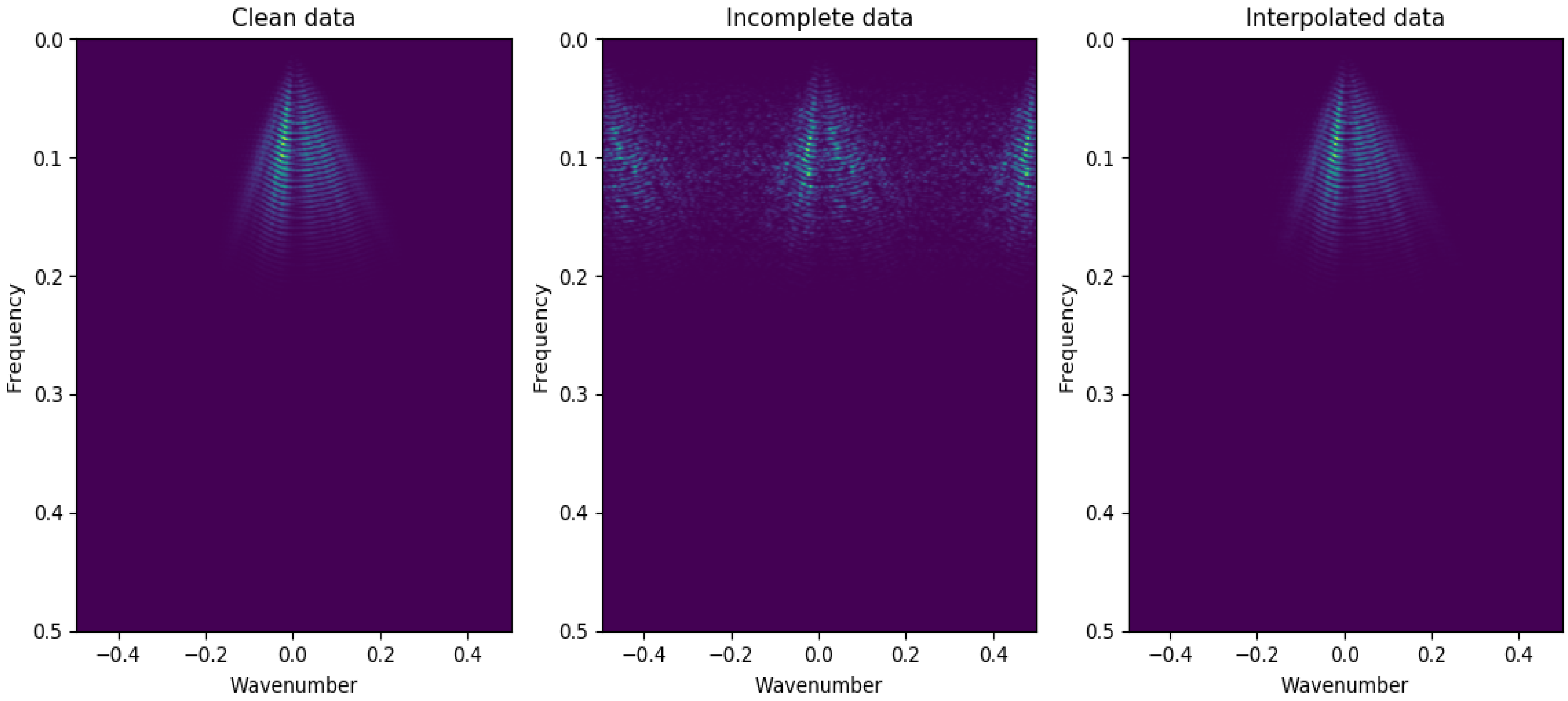


Clean data
Incomplete data
Interpolated data
Frequency
Wavenumber
0.0
0.1
0.2
0.3
0.4
0.5
−0.4
−0.2
0.0
0.2
0.4

2D Synthetic example: Curved data



F-K spectra



Clean data
Incomplete data
Interpolated data
Frequency
Wavenumber
0.0
0.1
0.2
0.3
0.4
0.5
−0.4
−0.2
0.0
0.2
0.4

3D Algorithm

Input: $\mathbf{dn}$, and max iteration K

Output: $\mathbf{dr}$

Initialization: $\mathbf{r}^{[0]} = \mathbf{dn}$,$\widehat{\mathbf{x}}^{[k]} = 0$, and $T^{[0]} = \emptyset$

for $k = 1, 2, ..., K$ **do**

$m = fftn(r)$

Finding the event with dip p and the angle q which has the maximum summation of the total energy in the $f - k$ domain.

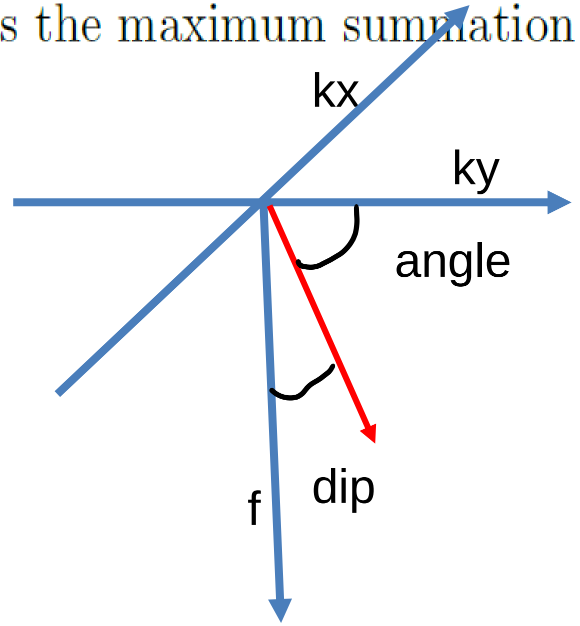
Picking all coefficients along the dip p,$\widehat{\mathbf{x}}^{[k]}$

$\widehat{\mathbf{x}}^{[k]}_{T^{[k]}} = \text{argmin}_{\widetilde{\mathbf{x}}_{T^{[k]}}} \|\mathbf{y} - ifftn(\widetilde{\mathbf{x}}_{T^{[k]}})\|_\rho$

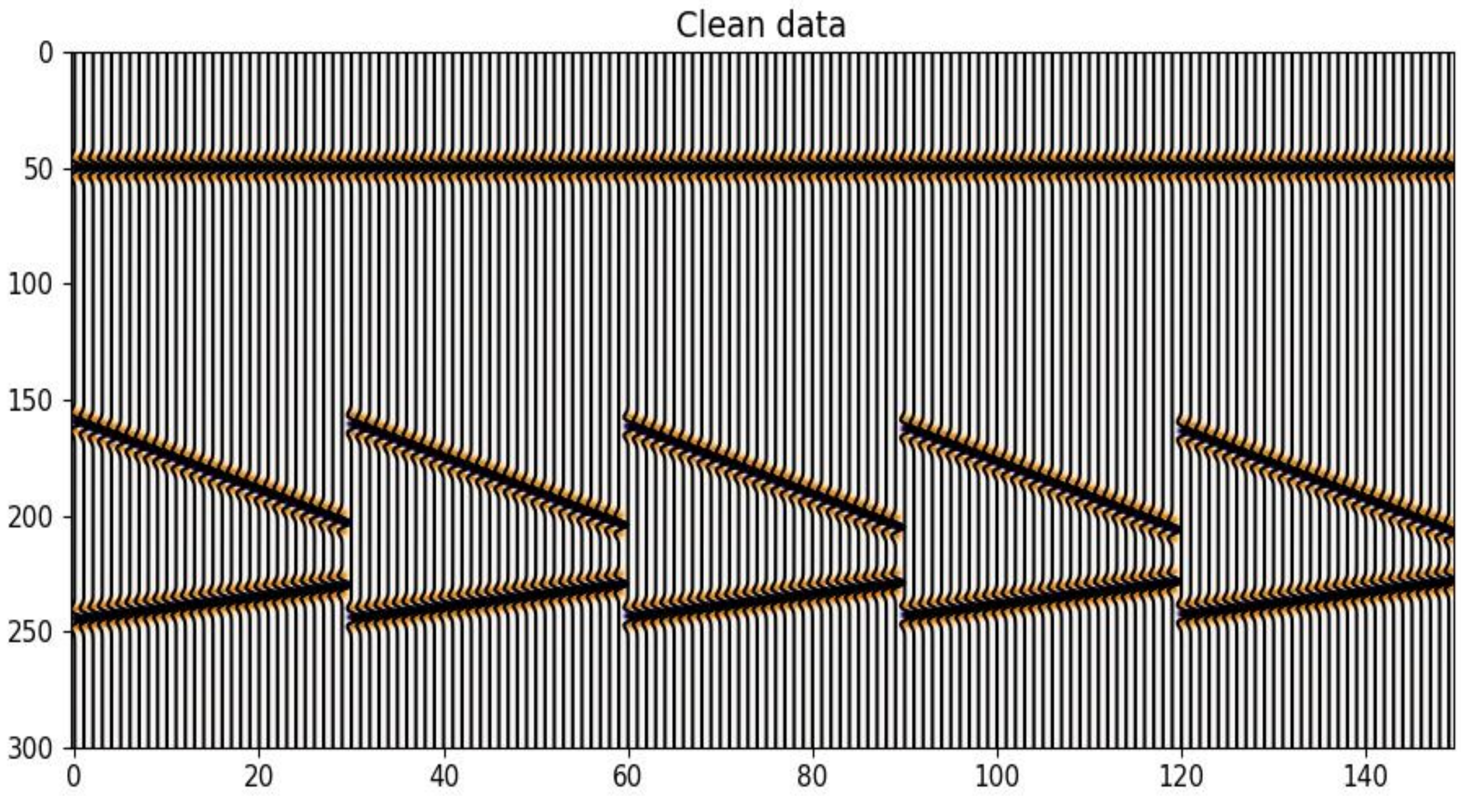
$\mathbf{r}^{[k]} = \mathbf{r} - ifftn(\widehat{\mathbf{x}}^{[k]}_{T^{[k]}})$

$\mathbf{dr}^{[k]} = \mathbf{dr} + ifftn(\widehat{\mathbf{x}}^{[k]}_{T^{[k]}})$

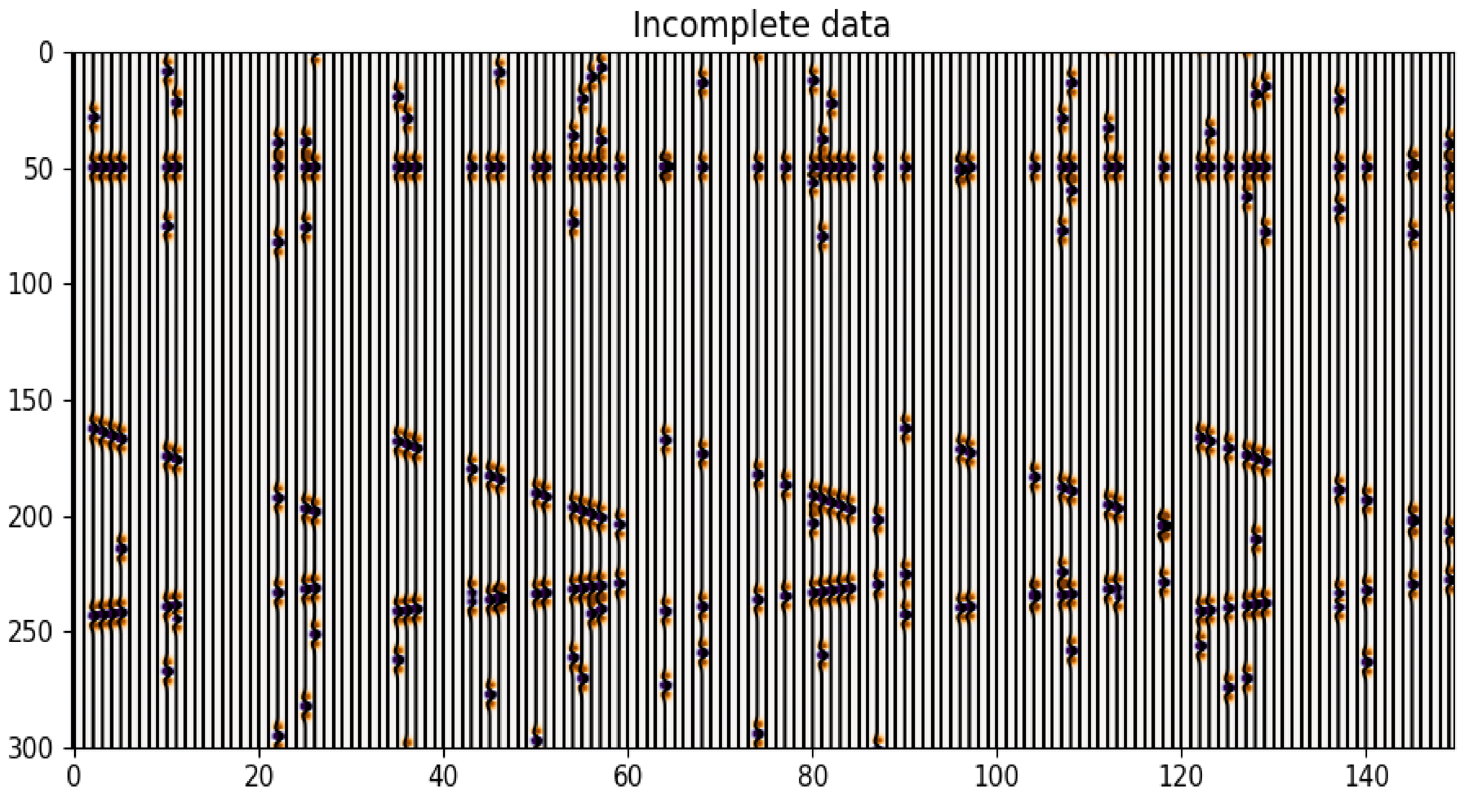
end for



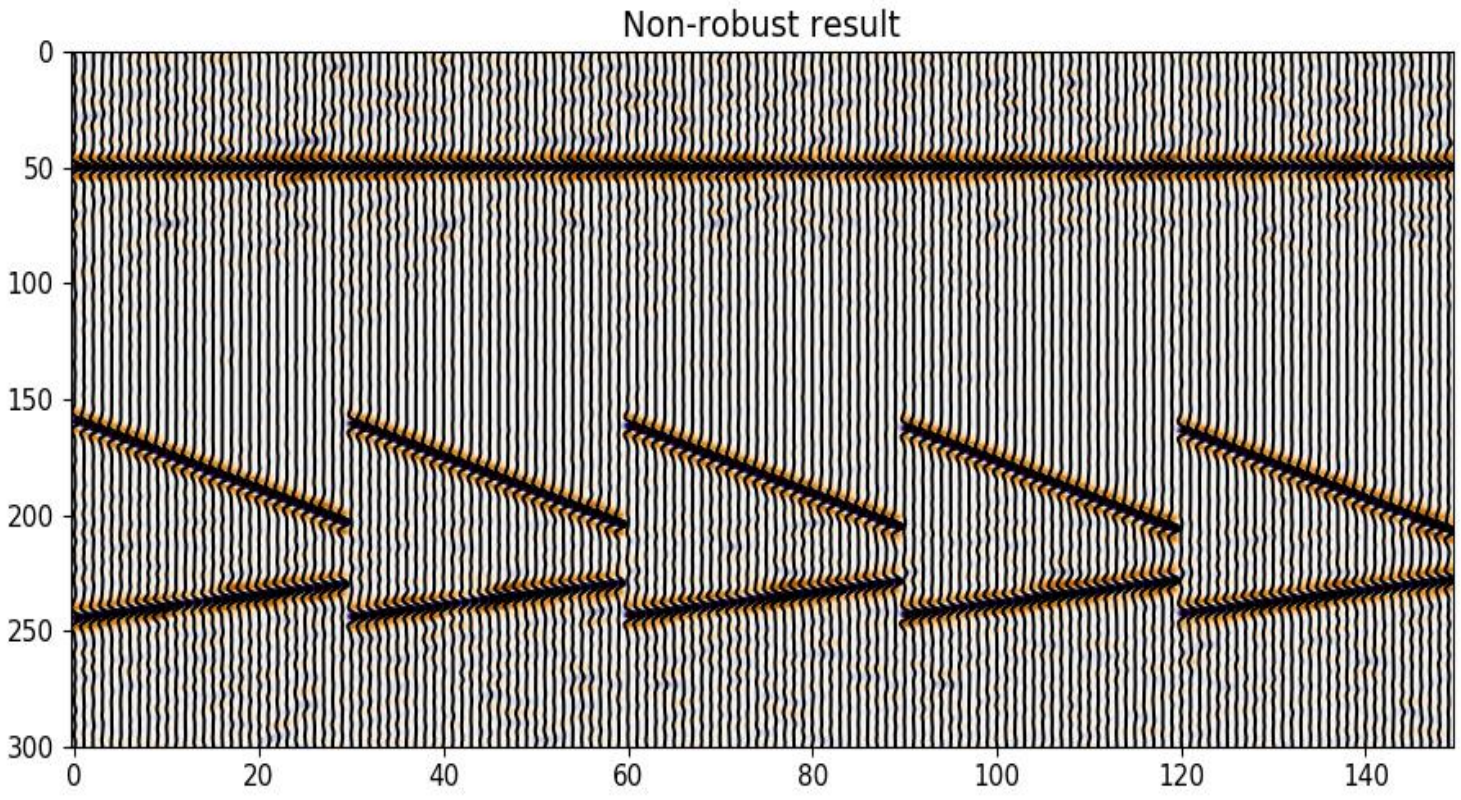
3D Example



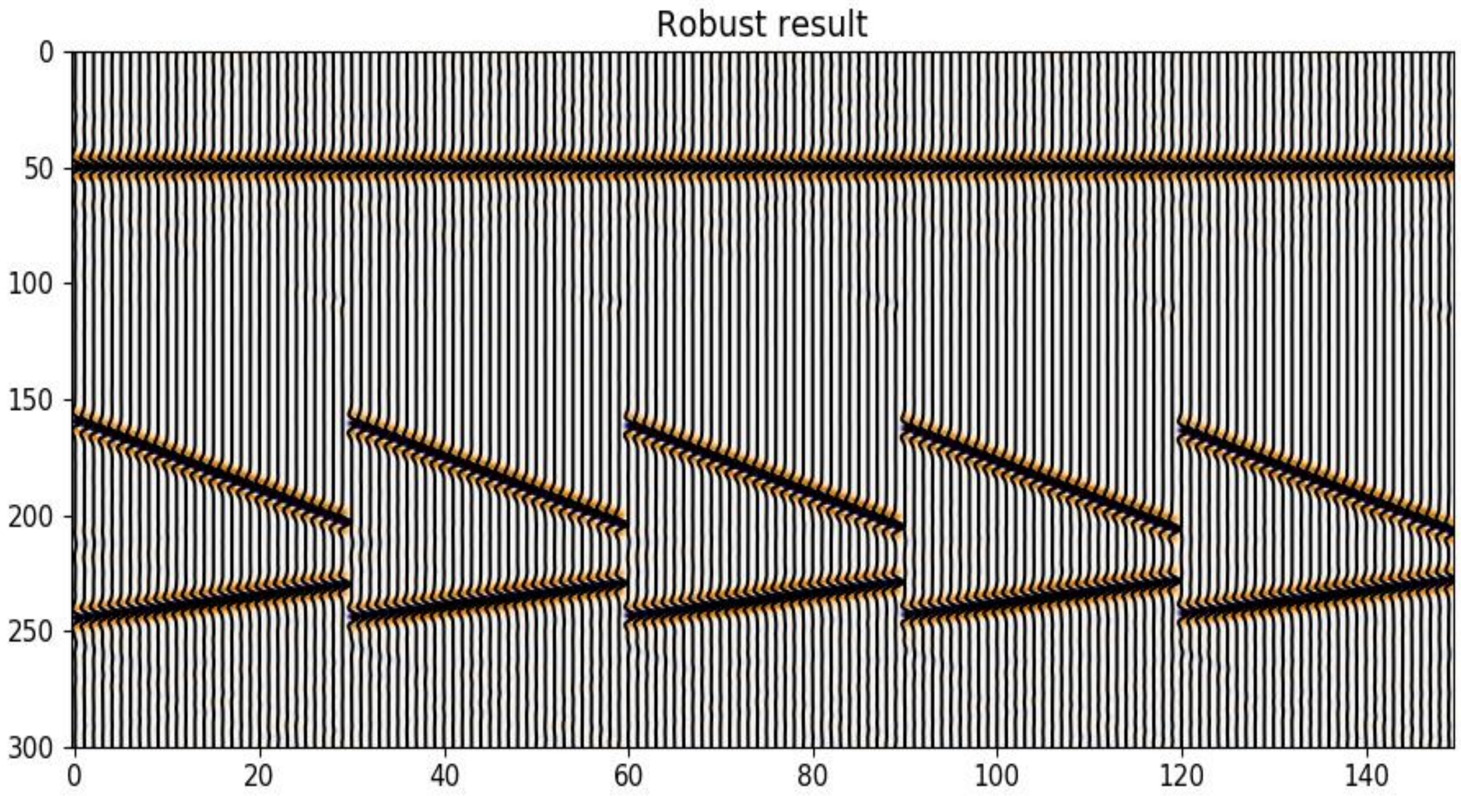
Incomplete data



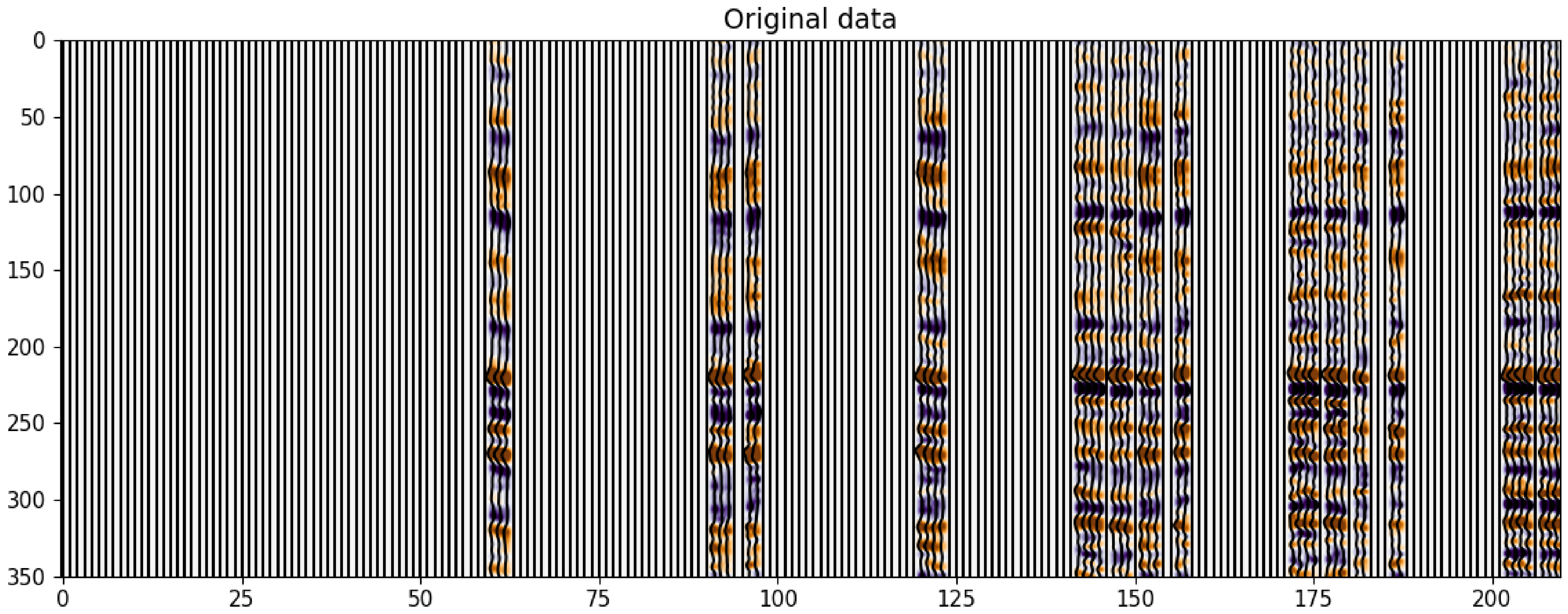
Non-robust result



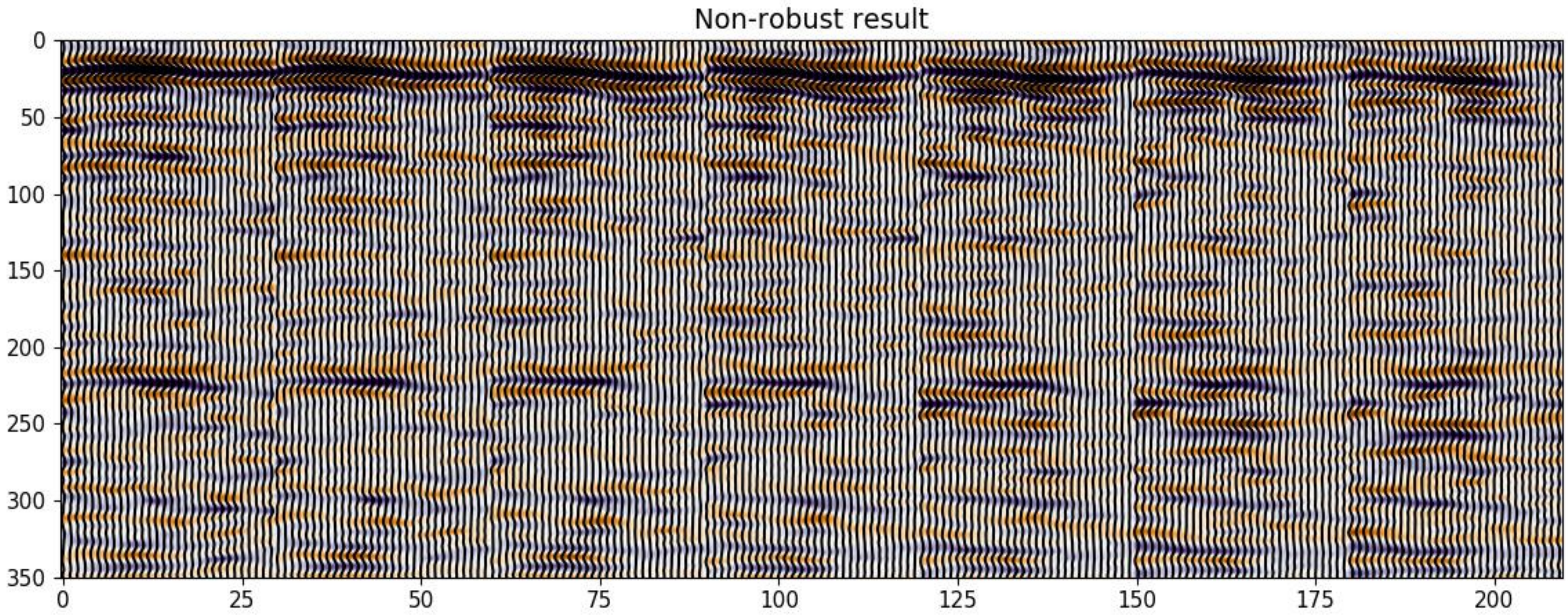
Robust result



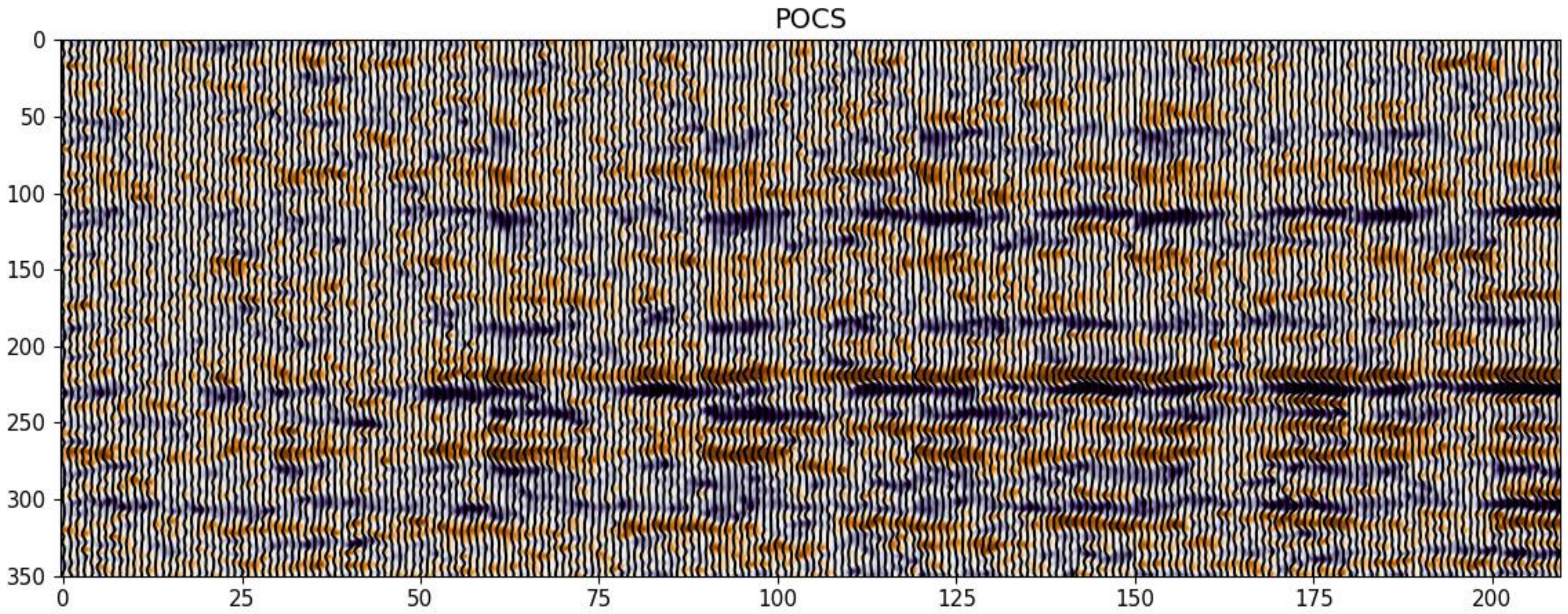
Real data


Original data
0
50
100
150
200
250
300
350
0
25
50
75
100
125
150
175
200

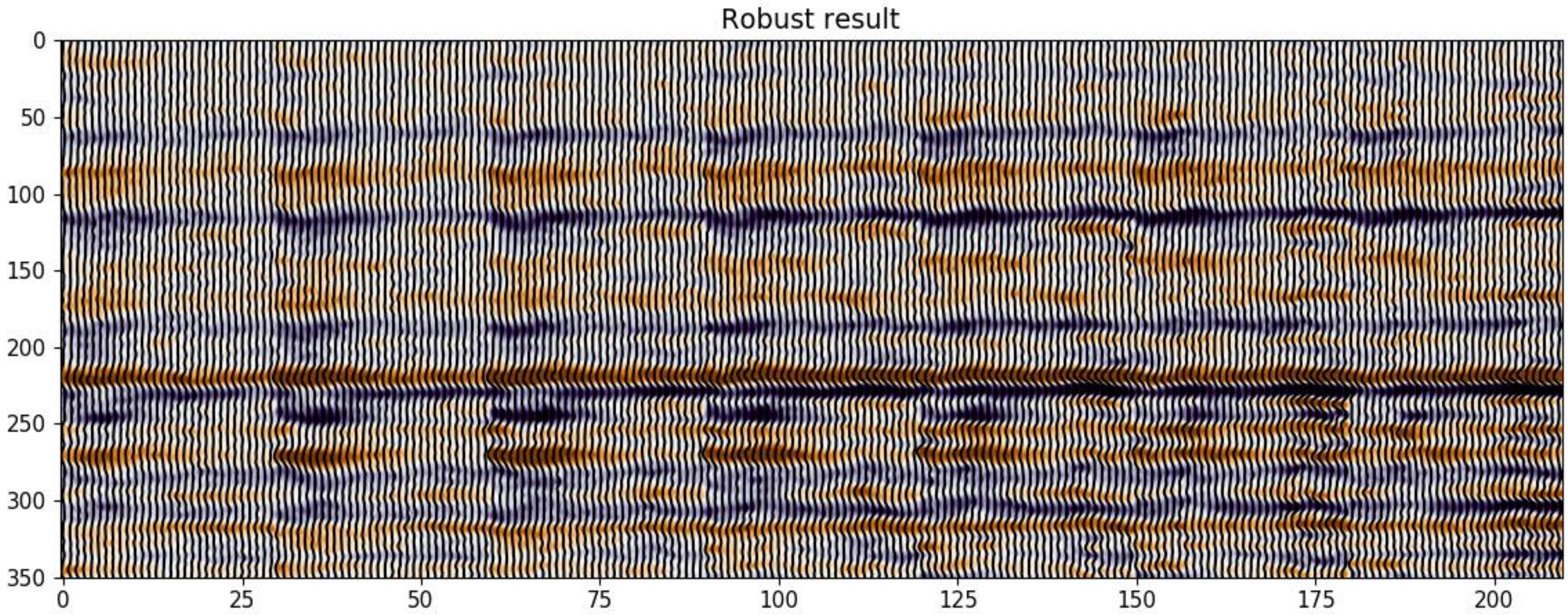
Non-robust result



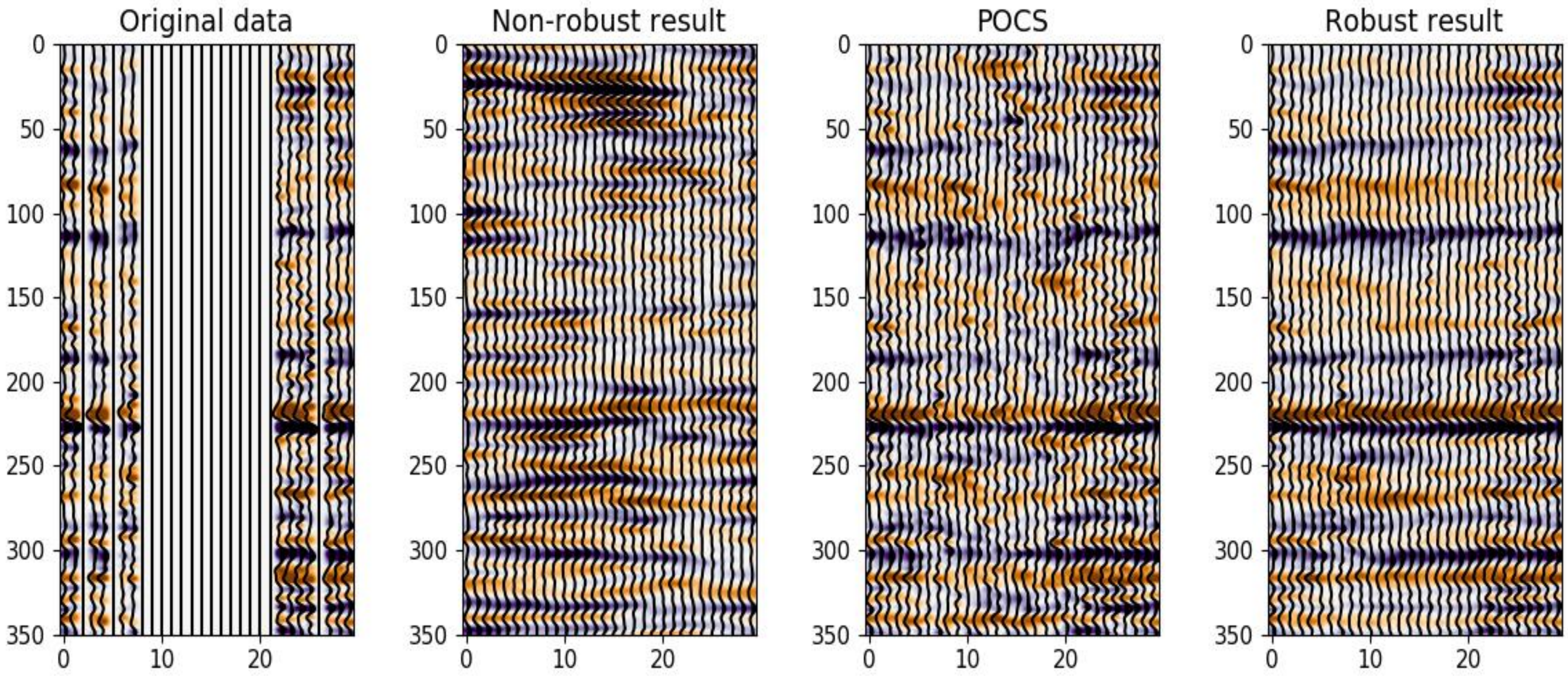
POCS result



Robust result



Real data example



Conclusions

- We proposed a new robust Orthogonal Matching Pursuit Fourier interpolation algorithm
- The robust OMPFI pick the dominant dips in the f-k domain and update the coefficients with robust M-estimators
- The results show that the proposed method can do interpolation and remove erratic noise simultaneous

Future work

- Extend to 5-D

Acknowledgments

- Sponsors of the Signal Analysis and Imaging Group at the University of Alberta