SIGNAL
ANALYSIS &
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A comparative study of seismic reconstruction for arbitrary irregular-grid acquisition: I-FMSSA vs. EPOCS

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2022. 01. 24

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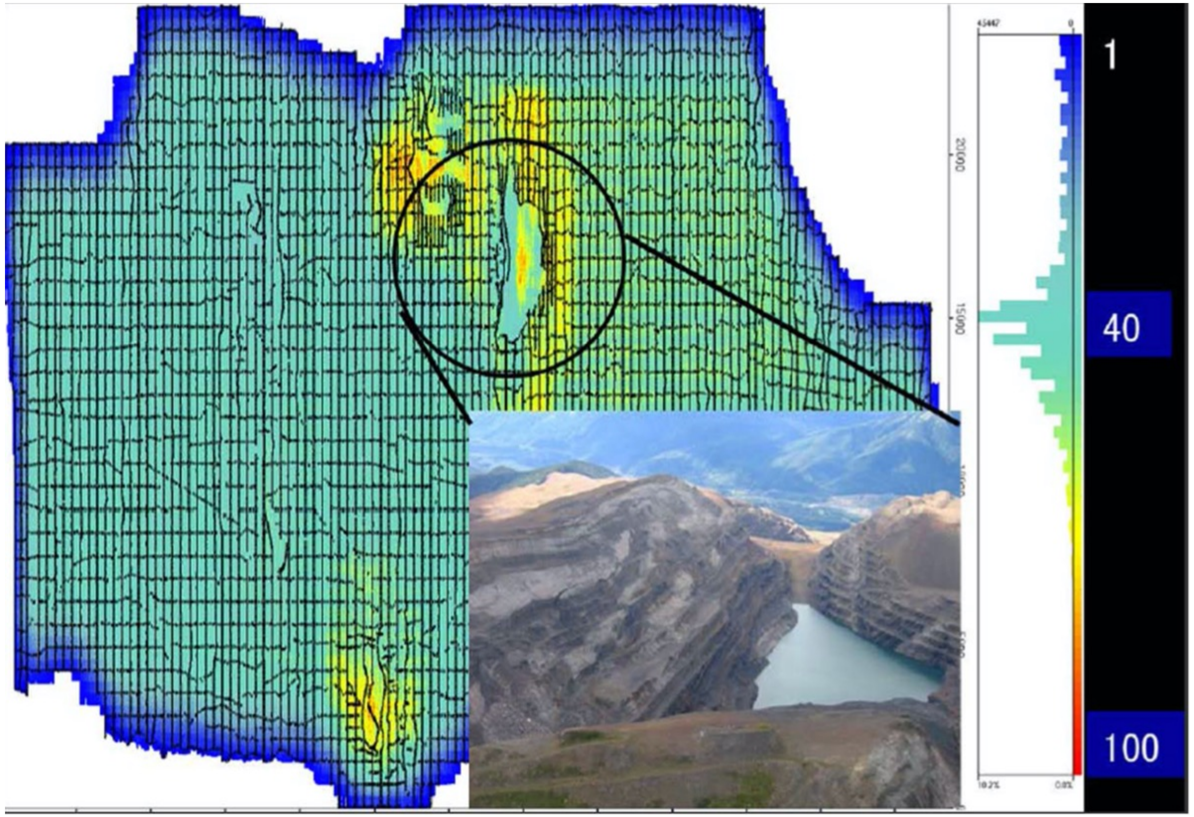
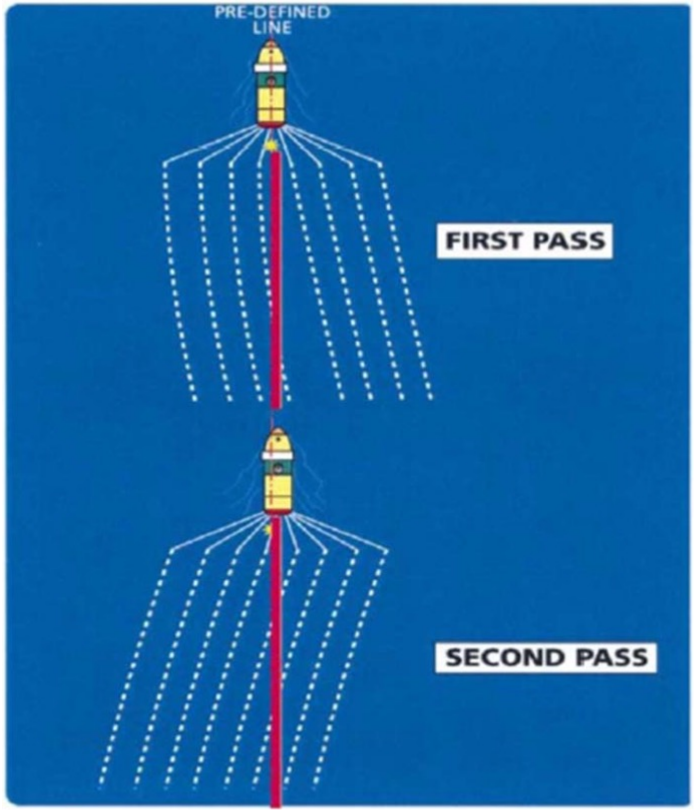
Introduction



(a) Irregularity caused by obstacles (Trad,2008).

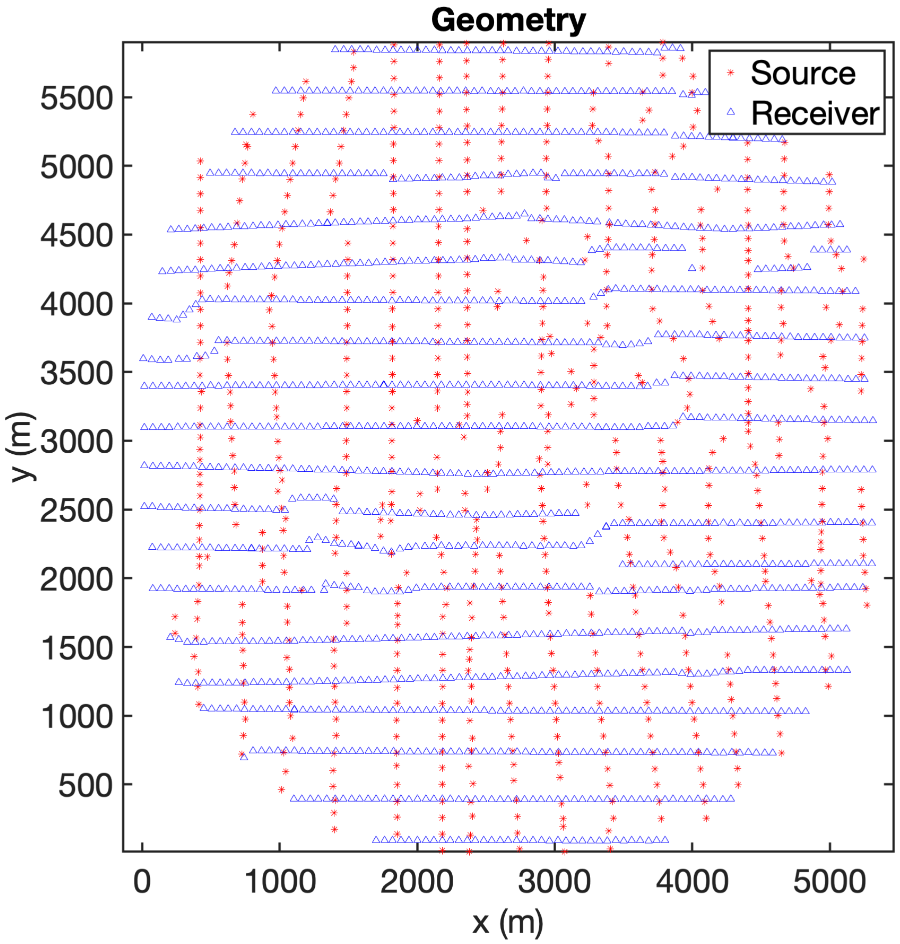


(b) Irregularity caused by feathering (Eiken et al.,2003).

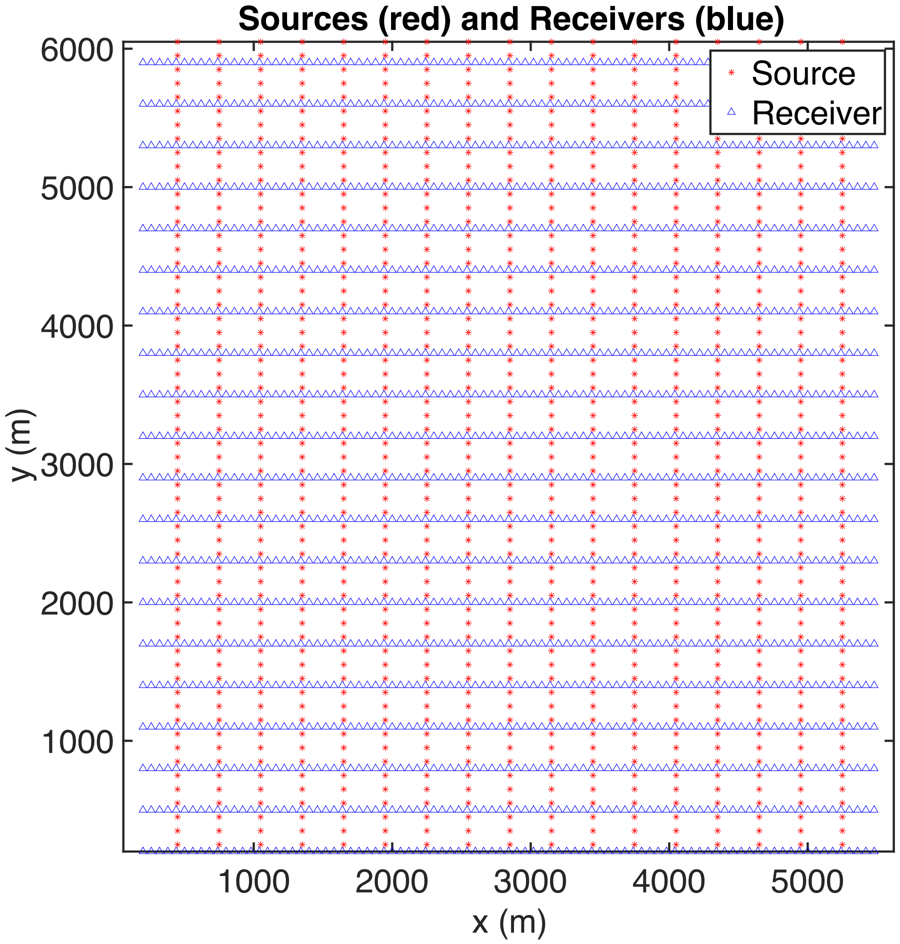
Many factors, such as **obstacles** and **featherings**, will lead to irregular distributions of seismic sources and receivers.

1. Natural observed irregularity

Observed irregular grid

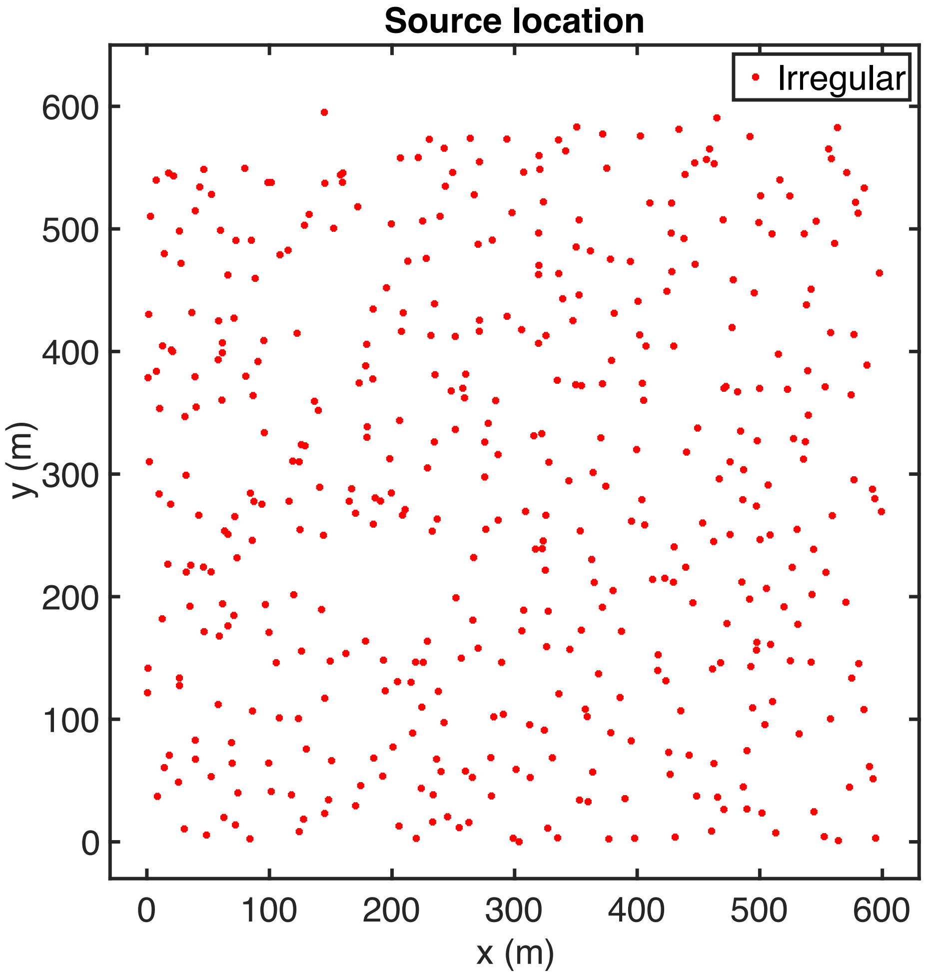


Desired regular grid

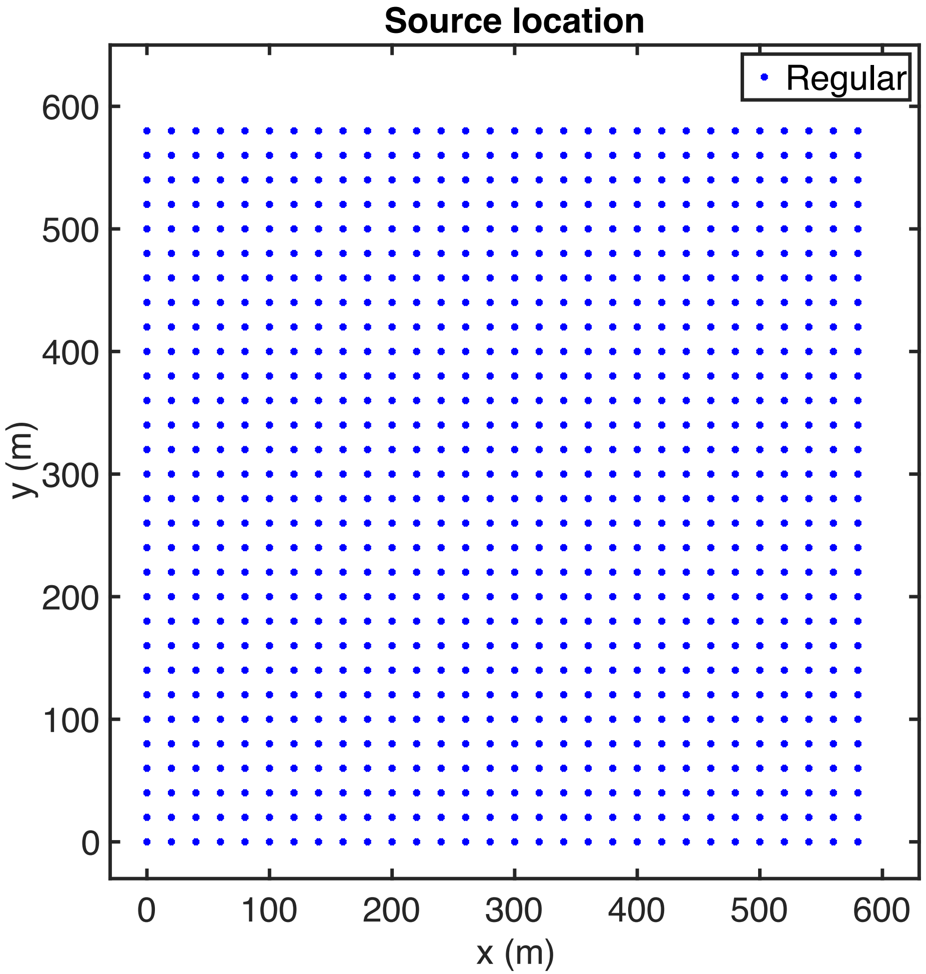


2. Human-made irregularity

CS-based irregular grid



Desired regular grid



Methods (EPOCS vs. I-FMSSA)

- ***POCS*** (Abma and Kabir, 2006)

- S is a promoting transform
- $\mathbf{T}_k$ is the iterative hard thresholding operator

$$\mathbf{d}_{k+1} = \mathbf{d}_{obs} + (\mathbf{I} - \mathscr{R})\mathcal{S}^T \mathbf{T}_k\left(\mathcal{S}\mathbf{d}_k\right)$$

Sampling operator

$$\mathscr{R}_{ij} = \begin{cases} 1 & \text{if one trace is assigned to grid point } (i,j) \\ 0 & \text{if grid point } (i,j) \text{ is empty} \end{cases}$$

- ***EPOCS*** (Jiang et al., 2017)

$$\mathbf{d}_{k+1} = \mathcal{W}^* \widehat{\mathbf{d}}_{obs} + (\mathbf{I} - \mathcal{W}^*\mathcal{W})\, \mathcal{S}^T \mathbf{T}_k\left(\mathcal{S}\mathbf{d}_k\right)$$

Interpolation operator

$$\mathcal{W}^* : \quad irregular \rightarrow regular$$

$$\mathcal{W} : \quad regular \rightarrow irregular$$

- ***Sinc-Kaiser interpolator***

I-FMSSA

- ***I-FMSSA*** (with low rank constraint) *(Carozzi and Sacchi, 2021)*

$$J = \|\mathbf{d}_{obs} - \mathcal{W}\mathbf{d}\|_2^2 \quad s.t. \quad rank(\mathbf{d}) \leq k$$

$$\mathbf{d}_{k+1} = \mathcal{P}[\mathbf{d}_k - \lambda\mathcal{W}^*(\mathcal{W}\mathbf{d}_k - \mathbf{d}_{obs})]$$

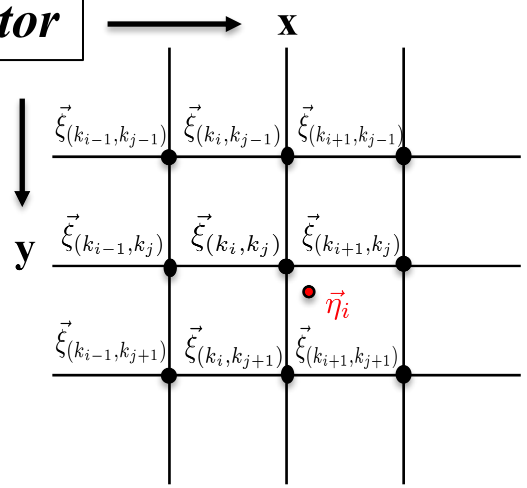
$\mathcal{W}^*$: *irregular* $\rightarrow$ *regular*

$\mathcal{W}$: *regular* $\rightarrow$ *irregular*

Projection operator = FMSSA (or MSSA)

- ***I-FMSSA:*** *Projection operator* = FMSSA
- ***I-MSSA:*** *Projection operator* = MSSA

- ***Interpolation operator***



Kaiser windowed ***sinc*** interpolation with N=1

$$\mathcal{W}_k(t) = \text{sinc}(\pi t)\frac{I_0\left(a\sqrt{1-(t/(N+1))^2}\right)}{I_0(a)}$$

FMSSA

• *MSSA*		• *FMSSA*
		(*Cheng et al. 2019*)
▪ Step 1: $d(x,t) \xrightarrow{\mathcal{F}(t)} D(f,t).$		
▪ Step 2: *Build Hankel matrix*	→	*Avoid Hankel structured matrices* • Hankel matrix vector products are computed via FFT
▪ Step 3: *Rank-reduction (SVD)*	→	*To speed up rank reduction* • RQRd is adopted to replace SVD.
▪ Step 4: *Anti-diagonal averaging*	→	*To improve anti-diagonal averaging* • Eigenimages are computed via convolutions.
▪ Step 5: $\hat{d}(x,t) \xleftarrow{\mathcal{F}^{-1}(t)} \hat{D}(f,t).$		

- Step 2: *Build Hankel matrix* → *Avoid Hankel structured matrices*
  - Hankel matrix vector products are computed via FFT

- Fast Hankel matrix vector product

➢ Hankel matrices can be embedded into circulant matrices

$$\mathbf{C} = \begin{bmatrix} D_2 & D_1 & D_3 \\ D_3 & D_2 & D_1 \\ D_1 & D_3 & D_2 \end{bmatrix} = \begin{bmatrix} D_3 & D_1 & D_2 \\ D_1 & D_2 & D_3 \\ D_2 & D_3 & D_1 \end{bmatrix} \begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}$$

➢ A Circulant matrix C multiplies a vector x are computed via FFT

$$\mathbf{Hx} = \begin{bmatrix} D_1 & D_2 \\ D_2 & D_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} D_2 & D_1 & D_3 \\ D_3 & D_2 & D_1 \\ D_1 & D_3 & D_2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ 0 \end{bmatrix} = \mathbf{C}\hat{\mathbf{x}} = \mathscr{F}^{-1}(\mathscr{F}(\mathbf{c}) \circ \mathscr{F}(\hat{\mathbf{x}}))$$

where $\mathbf{c} = \begin{bmatrix} D_2 & D_3 & D_1 \end{bmatrix}^T$ and $\hat{\mathbf{x}} = \begin{bmatrix} x_2 & x_1 & 0 \end{bmatrix}^T$

(*Cheng et al. 2019*)

- Step 3: *Rank-reduction (SVD)* → *To speed up rank reduction*
  - RQRd is adopted to replace SVD.

- Randomized QR decomposition (RQRd)

1. Projection onto random sets:

$$\underset{M\times p}{X} = \underset{M\times L}{H}\ \underset{L\times p}{\Omega}$$

Ω are *p* random vectors, and *p* << *L*

2. Economic-size QR decomposition:

$$\underset{M\times p}{Q}\ \underset{p\times p}{R} = qr(\underset{M\times p}{X})$$

3. Low-rank approximation:

$$\hat{\mathbf{H}} = \mathbf{Q}\mathbf{Q}^{\mathbf{H}}\mathbf{H}$$

(*Cheng et al. 2019*)

- Step 4: *Anti-diagonal averaging* → *Fast anti-diagonal averaging*
  - Eigenimages are computed via convolutions.

- Fast anti-diagonal averaging

For one linear event (rank=1 case)

➢ Let *p* = *1*: $\hat{\mathbf{H}} = \mathbf{q}_1\mathbf{q}_1^H\mathbf{H}$

Let $\mathbf{t}_1 = \mathbf{q}_1^H\mathbf{H}$, we get $\hat{\mathbf{H}} = \mathbf{q}_1\mathbf{t}_1$

➢ For **anti-diagonal averaging**:

$$\hat{\mathbf{D}} = \begin{cases} \frac{1}{i}\sum_{j=1}^{i} q_{1j}t_{1-j+1}, & 1 \le i \le M \\ \frac{1}{M}\sum_{j=1}^{M} q_{1j}t_{1_{i-j+1}}, & M \le i \le L \\ \frac{1}{N-i+1}\sum_{j=i-L+1}^{M} q_{1j}t_{1_{i-j+1}}, & L \le i \le N \end{cases} = w\sum_{j=1}^{N} q_{1_j}t_{1_{i-j+1}}$$

Convolution

➢ For rank = *p*

$$\hat{\mathbf{D}} = w \circ [(\mathbf{q}_1 * \mathbf{t}_1) + (\mathbf{q}_2 * \mathbf{t}_2) + \cdots + (\mathbf{q}_p * \mathbf{t}_p)]$$

(*Cheng et al. 2019*)

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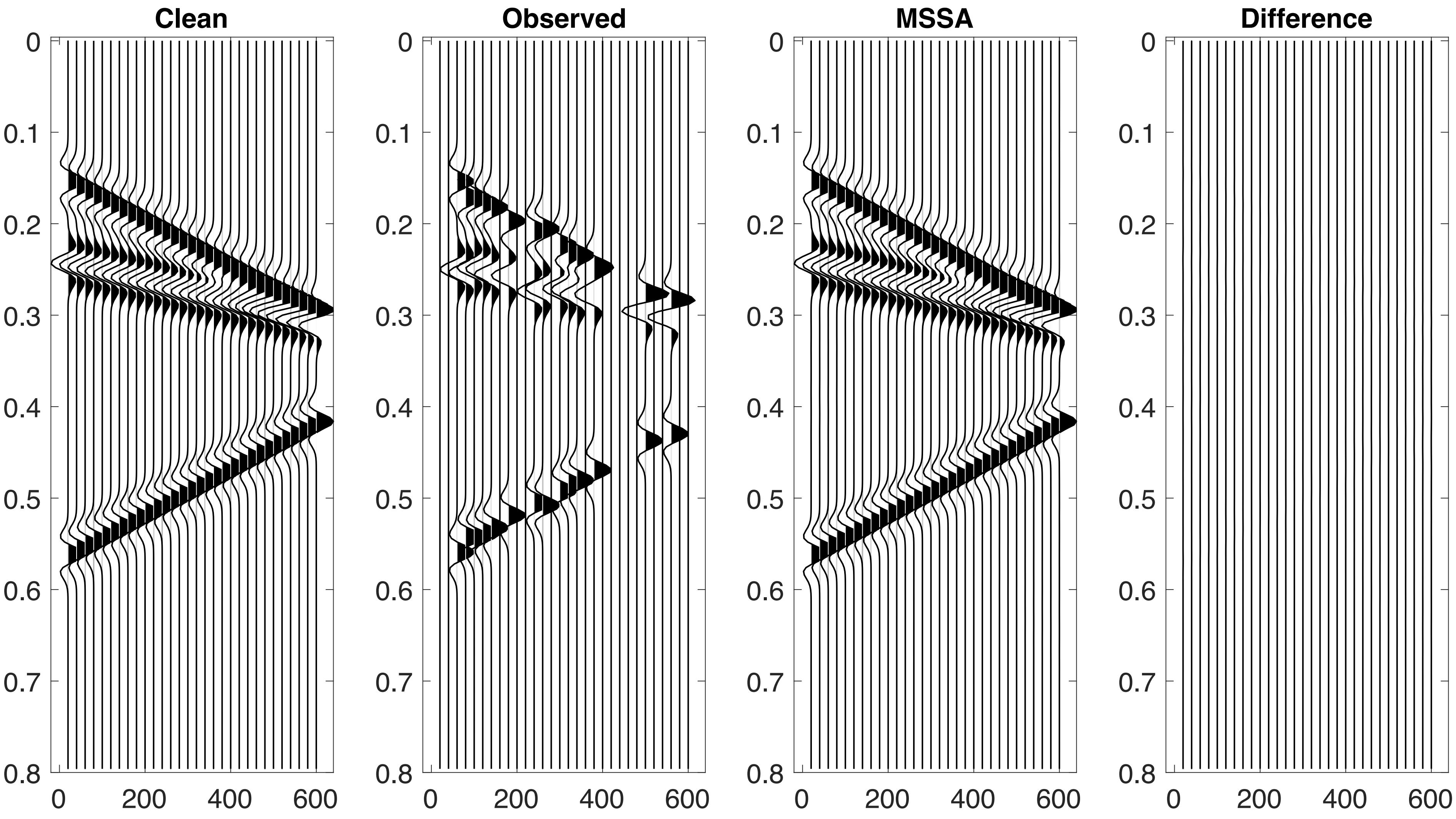
I-MSSA vs. I-FMSSA (Computational efficiency?)

3D Reconstruction I-FMSSA Vs I-MSSA

- ***I-MSSA***

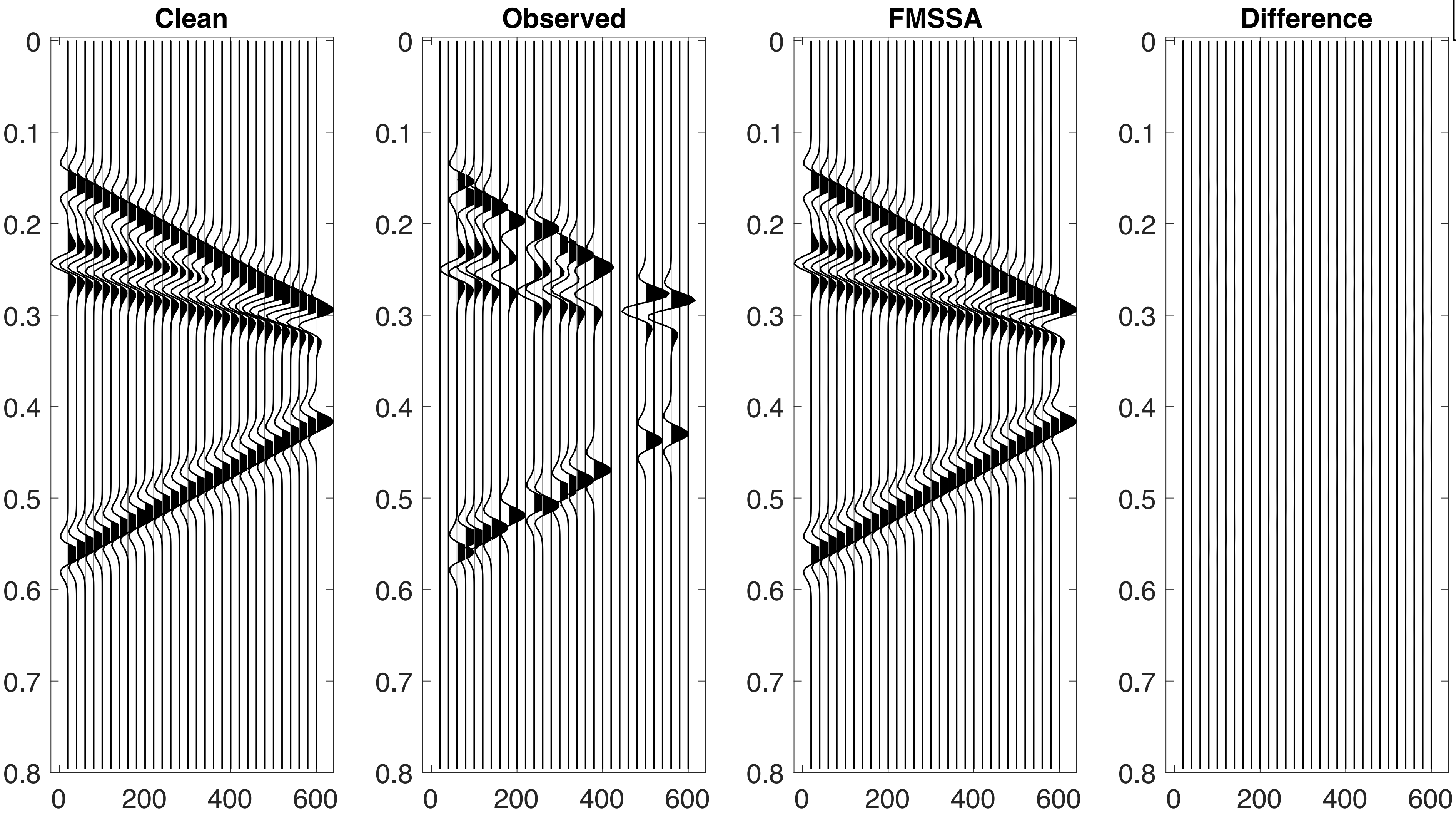
Data size: 200x30x30
Time = 31.92 s
SNR = 45.9 dB



3D Reconstruction I-FMSSA Vs I-MSSA

- ***I-FMSSA***

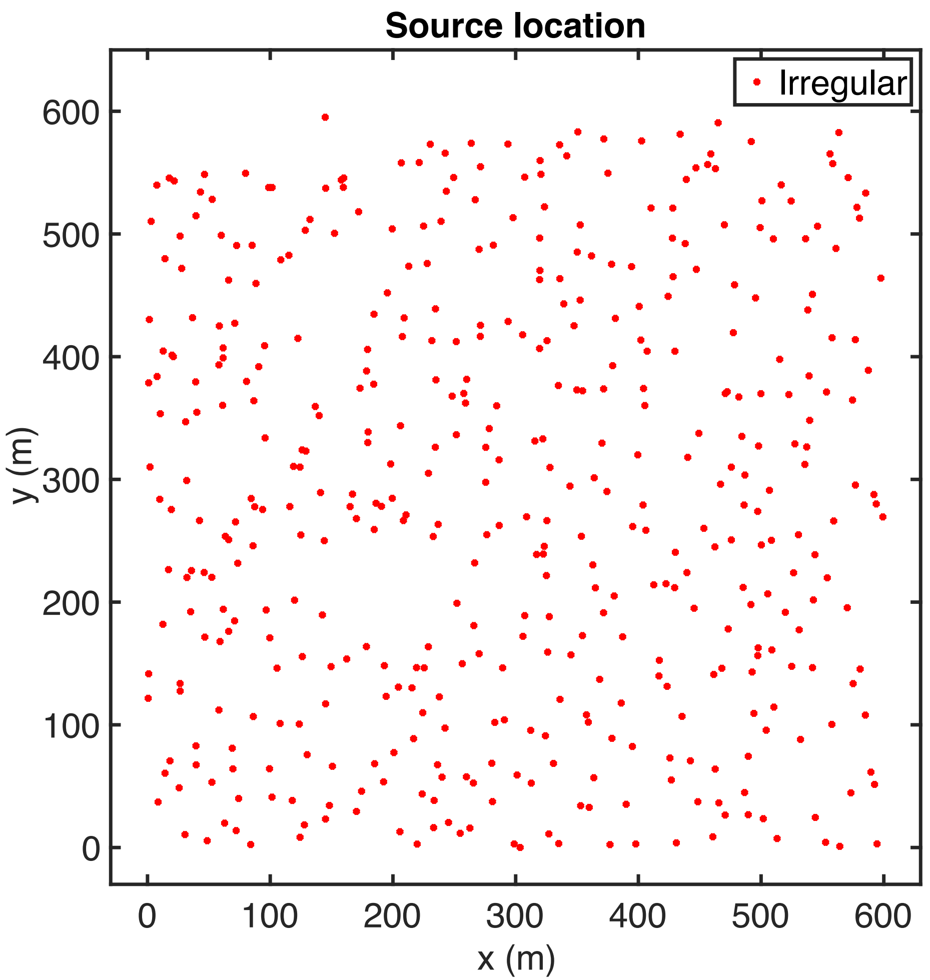
Data size: 200x30x30
Time = 7.71 s
SNR = 46.7 dB



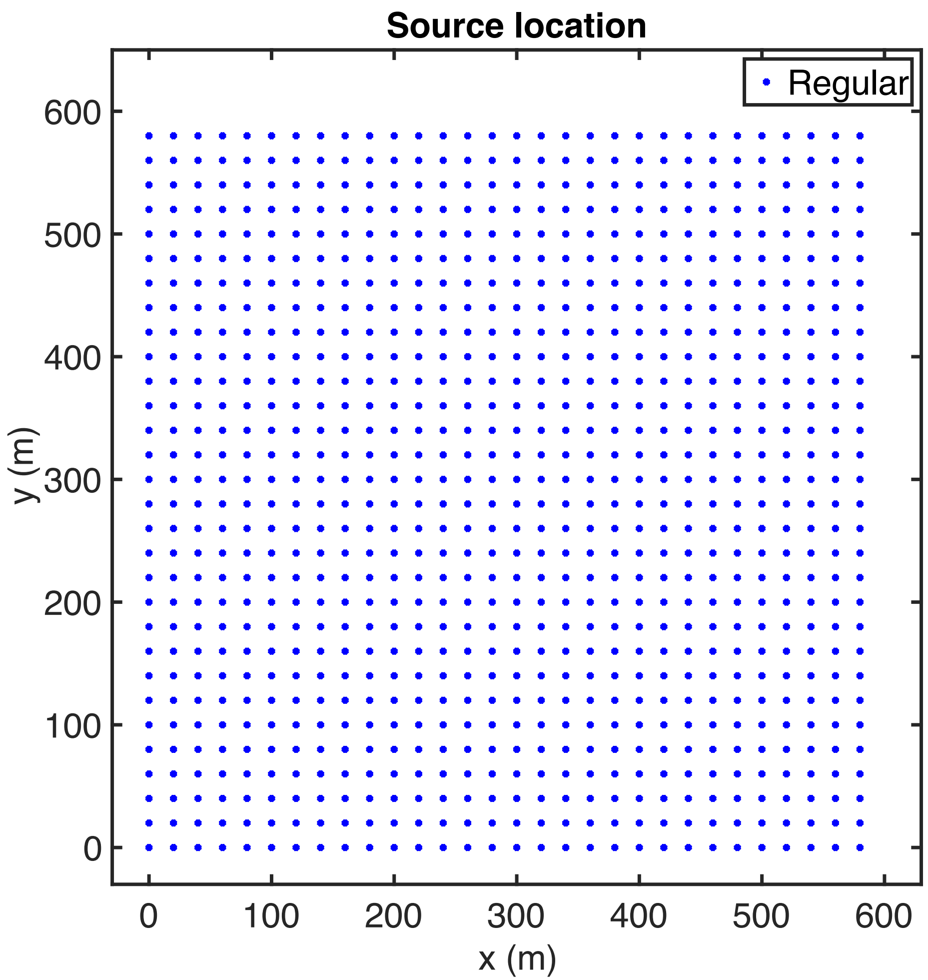
Synthetic Example (EPOCS vs. I-FMSSA)

Synthetic Example

- Geometry of source location

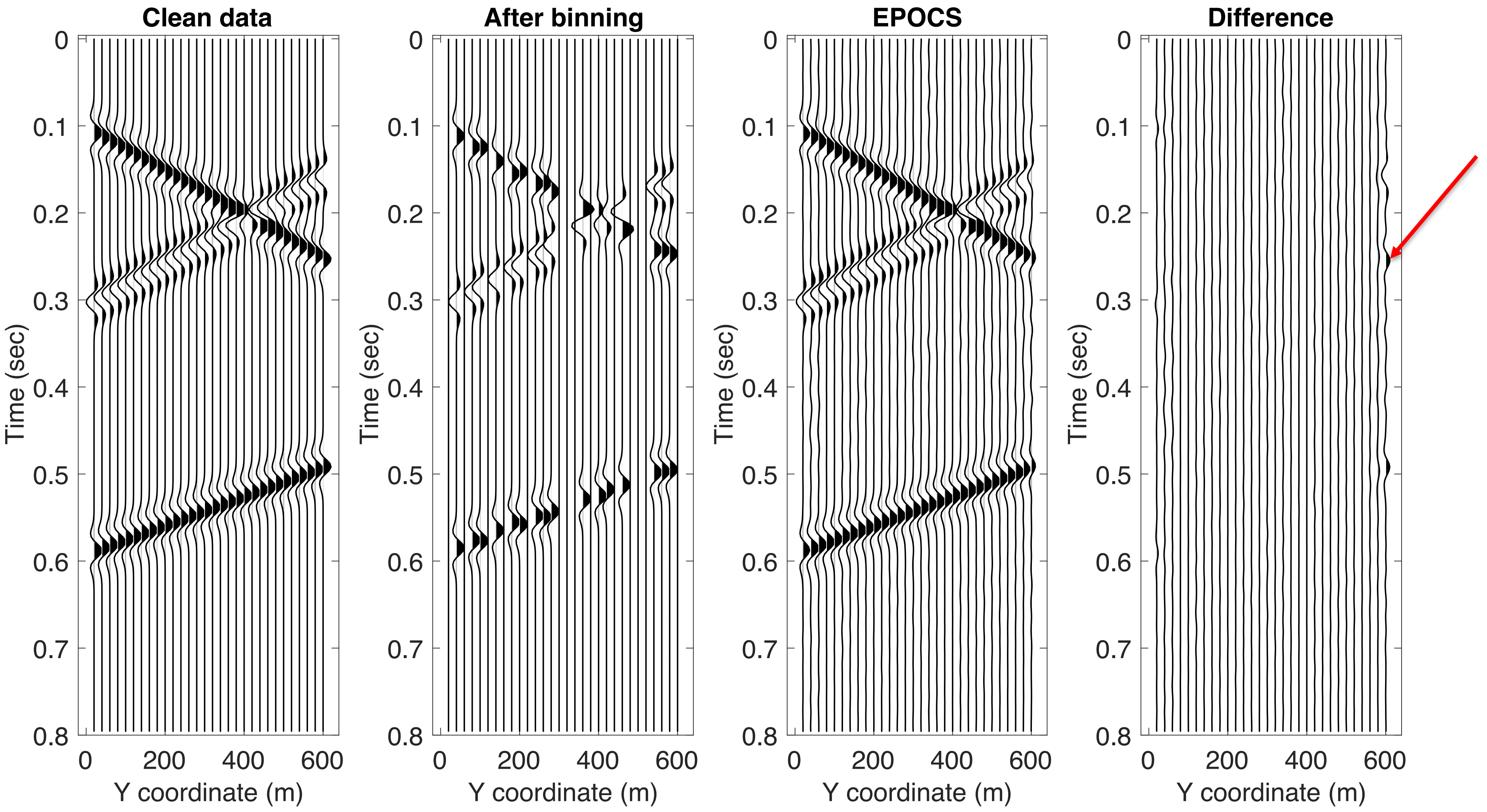


50% decimation



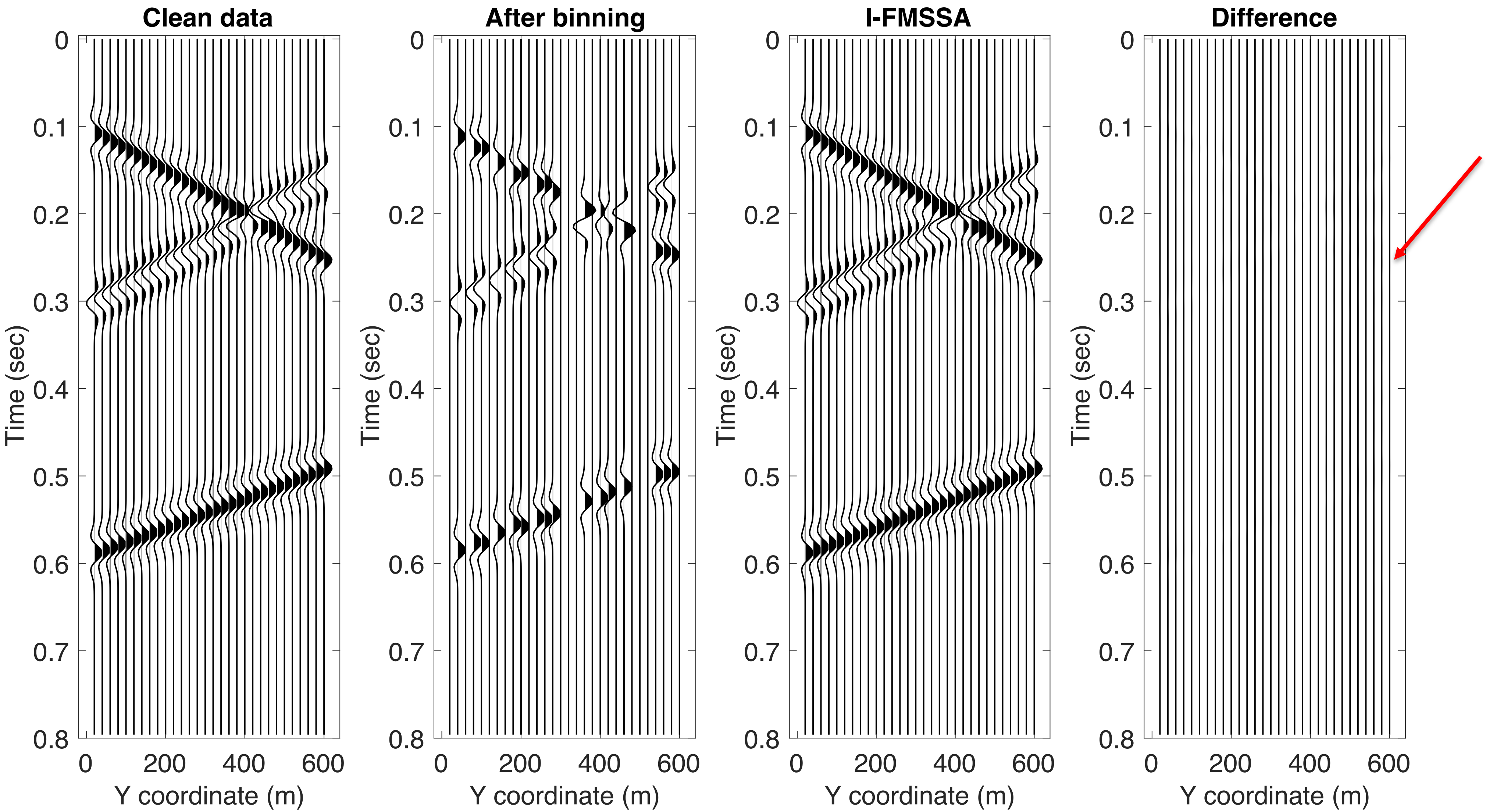
Synthetic Example

- EPOCS **d**(:,13,:)



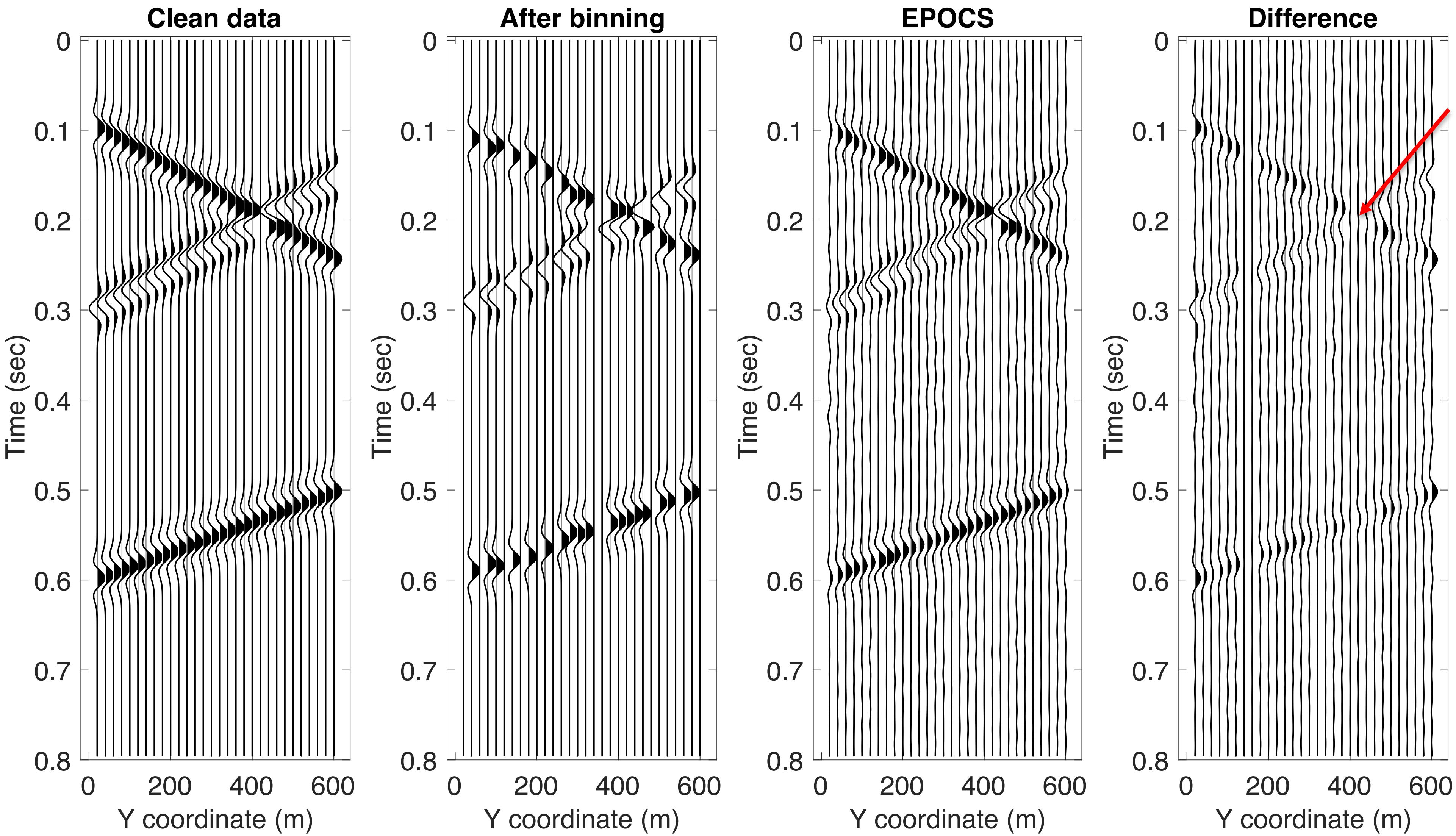
Synthetic Example

- I-FMSSA $\mathbf{d}(:,13,:)$



Synthetic Example (Boundary effect analysis)

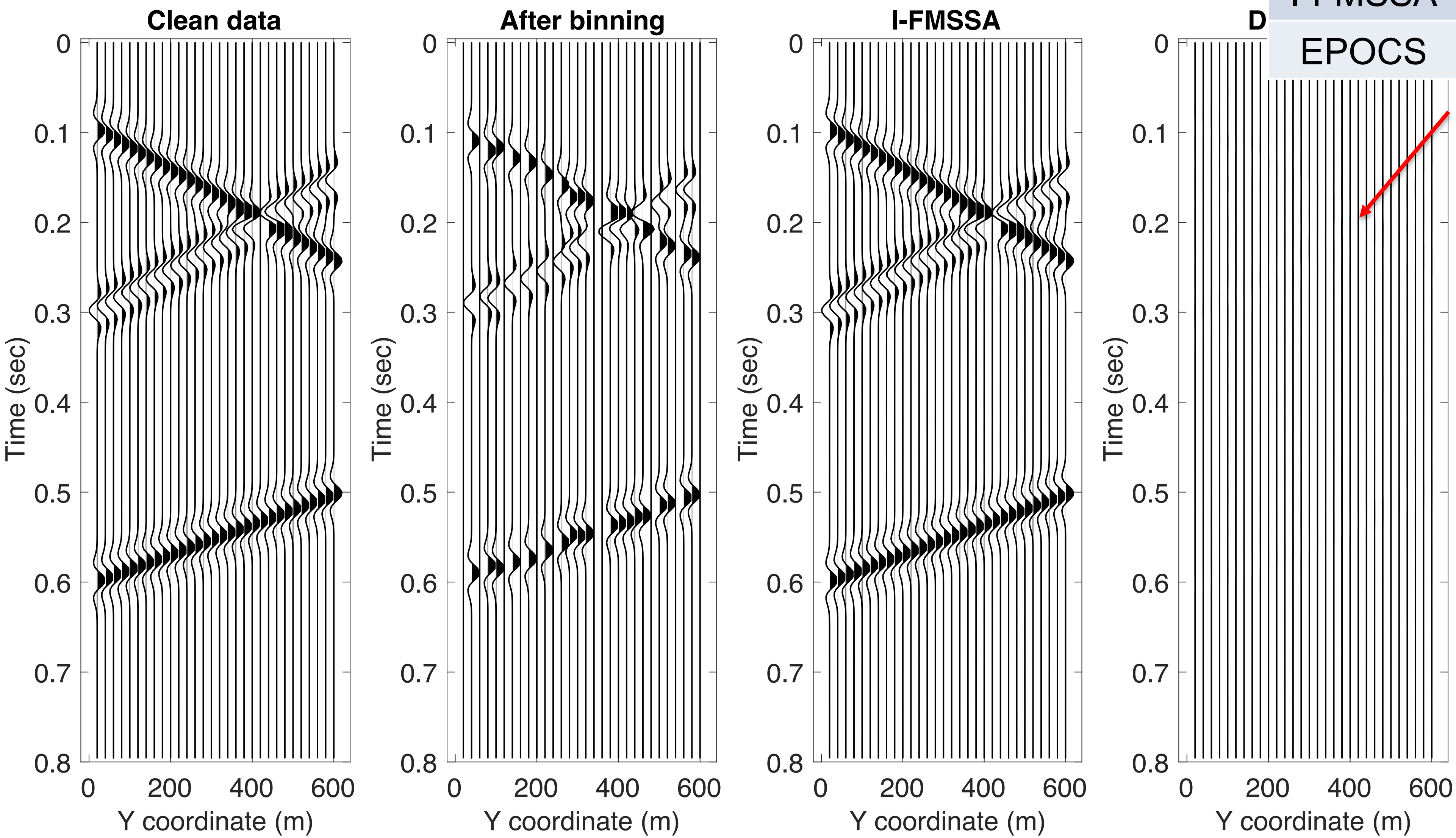
- EPOCS $\mathbf{d}(:,1,:)$



Synthetic Example (Boundary effect analysis)

- I-FMSSA $\mathbf{d}(:,1,:)$

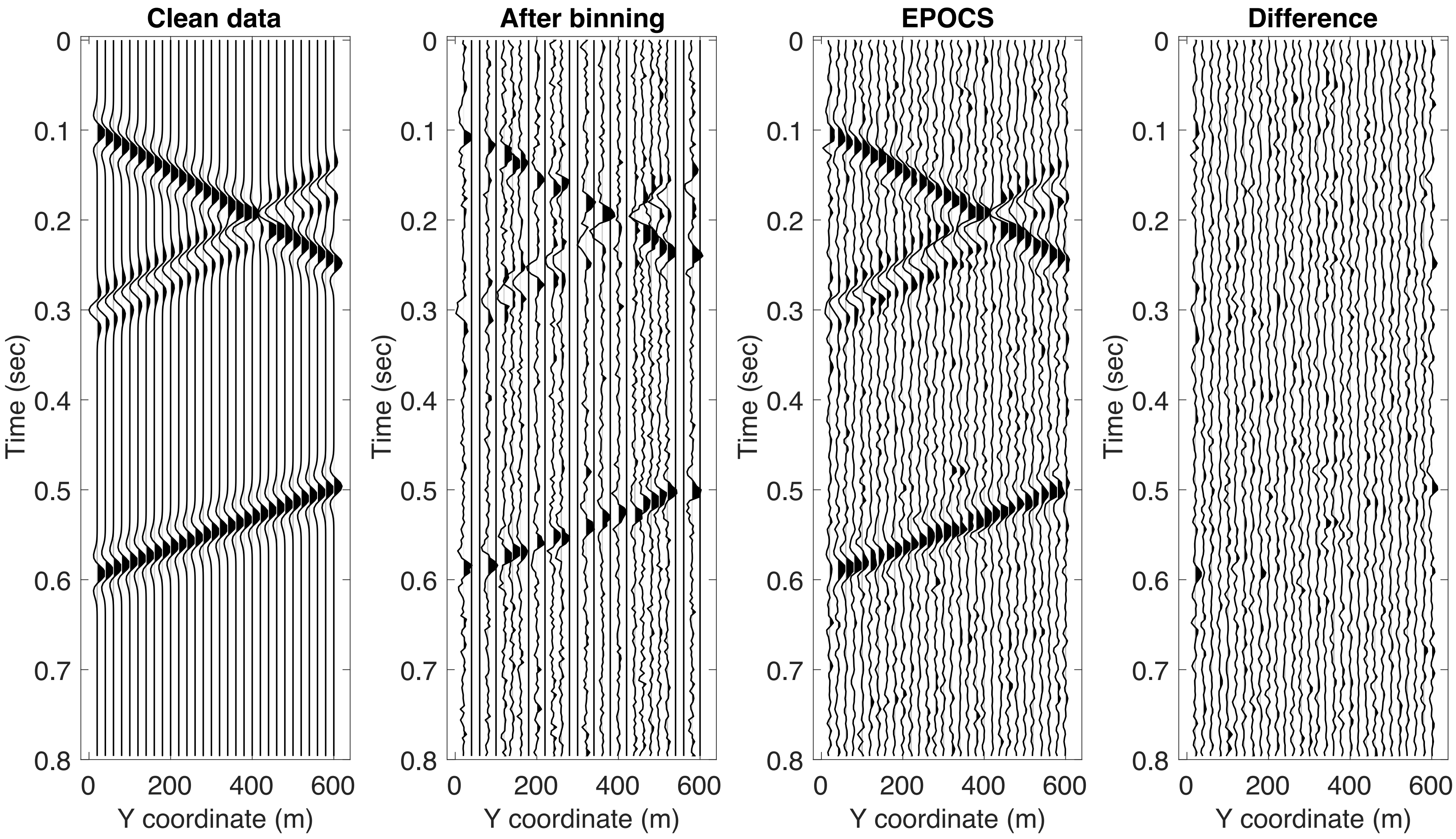
	SNR (dB)
I-FMSSA	47.30
EPOCS	16.18



Synthetic Example (with random noise)

Synthetic Example

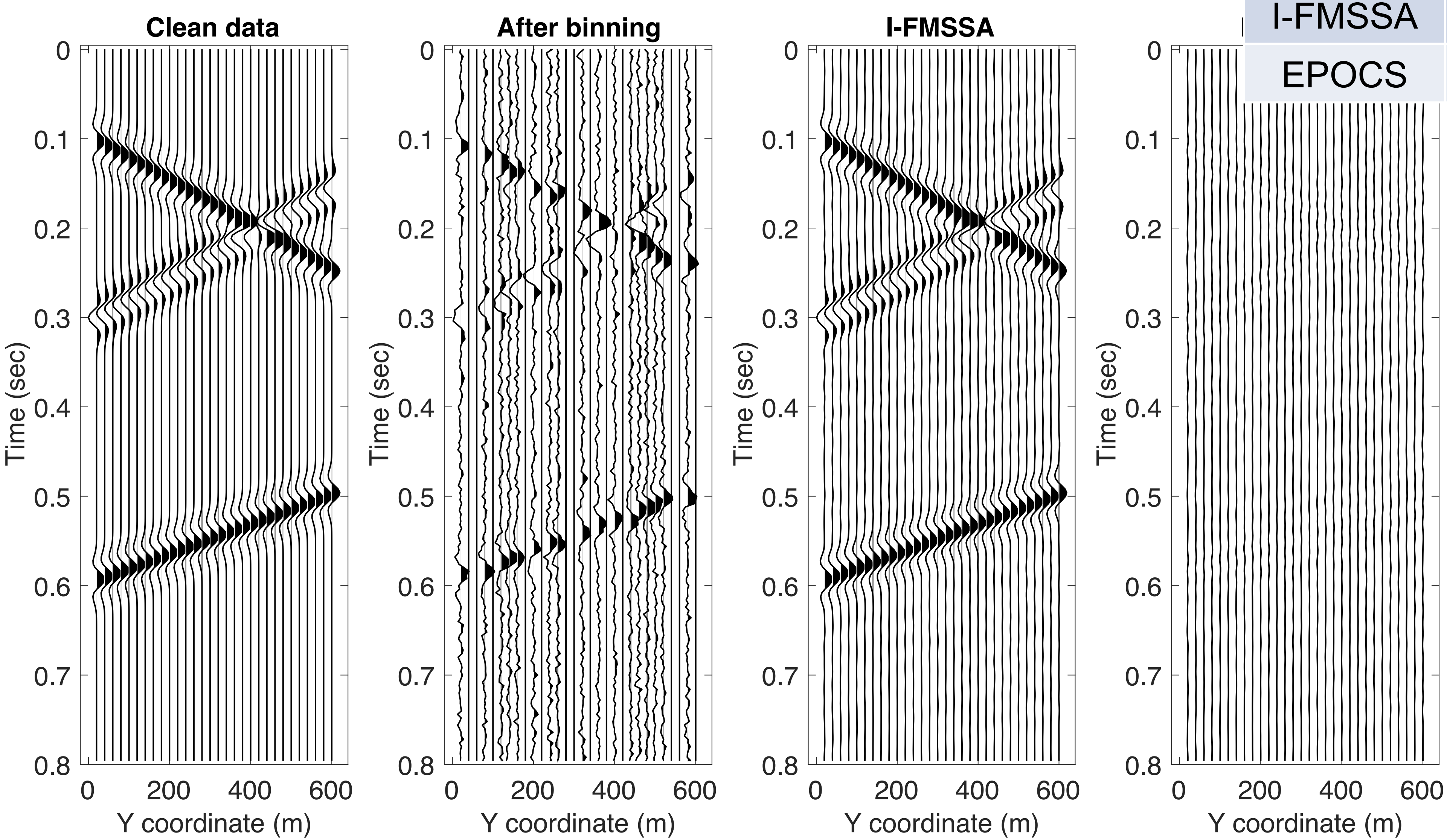
- EPOCS $\mathbf{d}(:,13,:)$



Synthetic Example

- I-FMSSA $\mathbf{d}(:,13,:)$

	SNR (dB)
I-FMSSA	20.74
EPOCS	3.44



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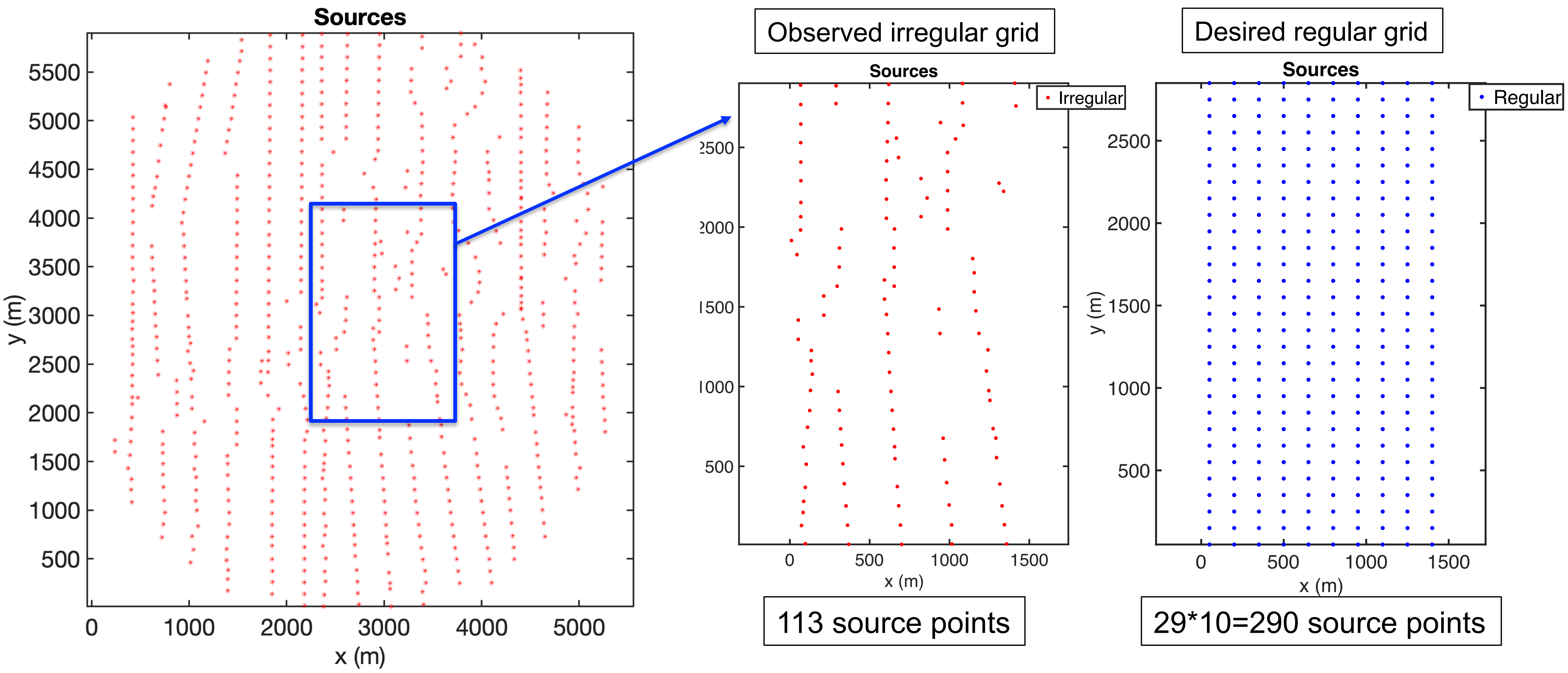
Real Example (Irregular Reconstruction)

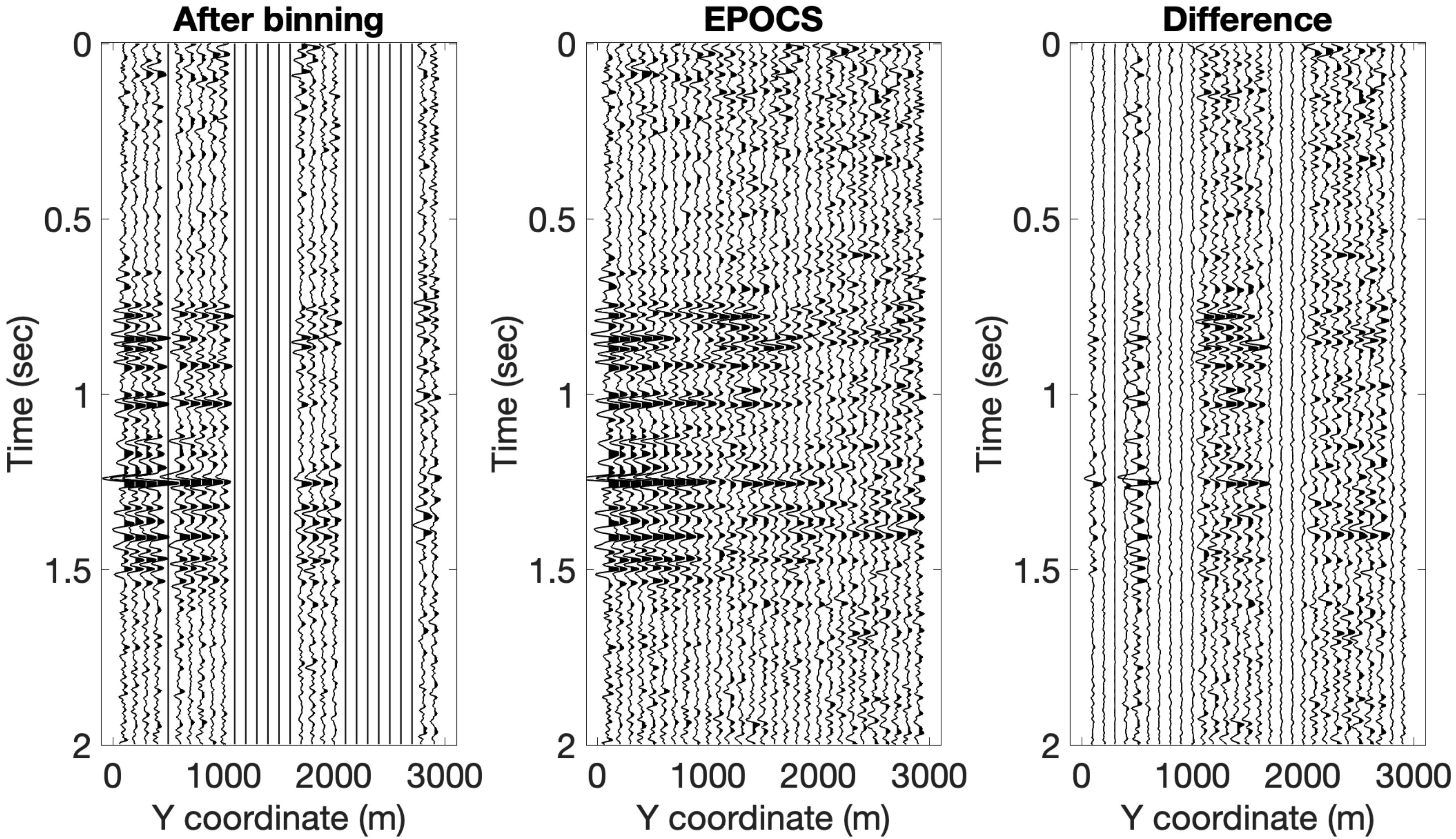
Real Example

- Geometry of source location



- EPOCS $\mathbf{d}(:,3,:)$

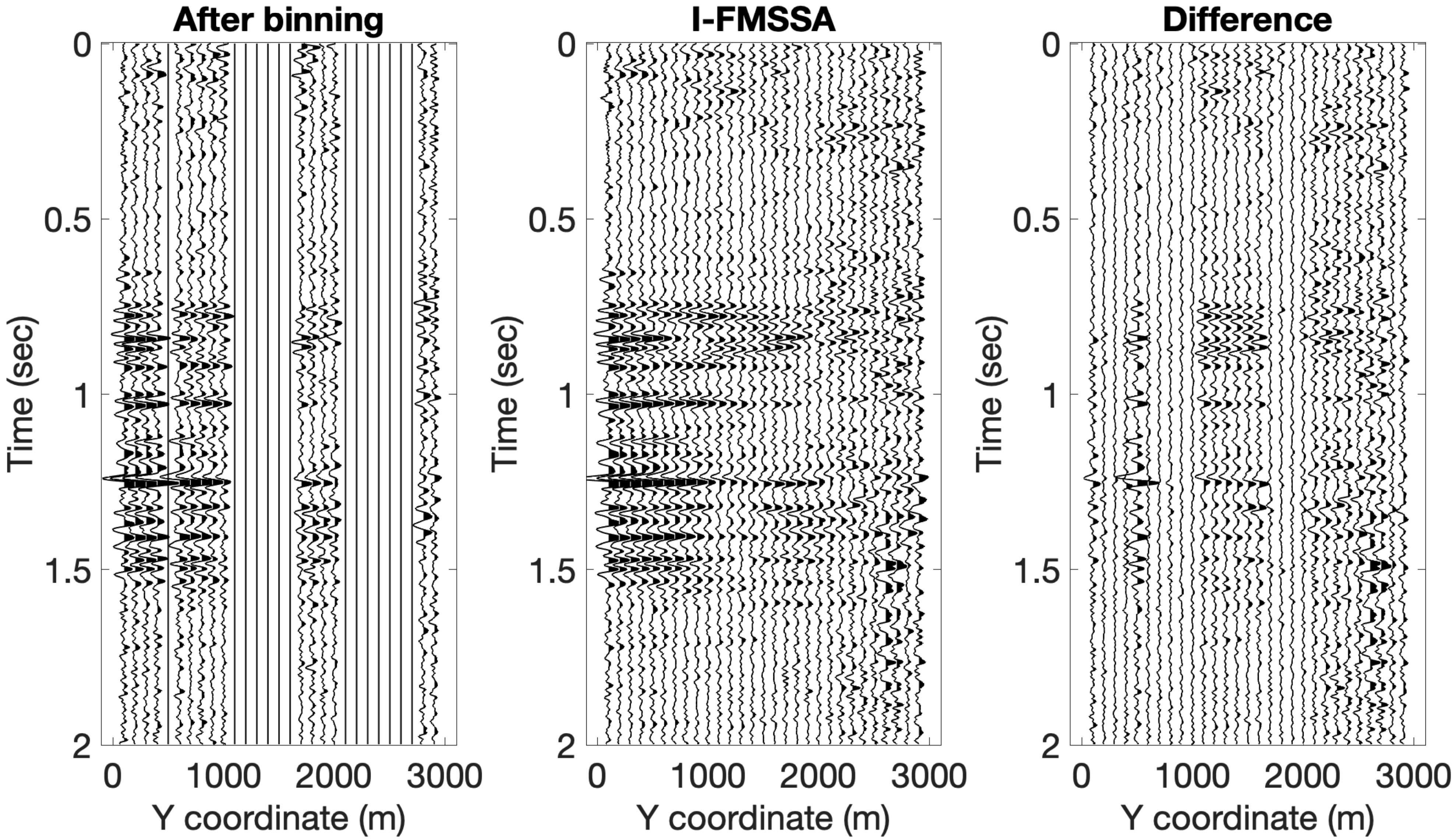
Inline slice



Real Example

- I-FMSSA $\mathbf{d}(:,3,:)$

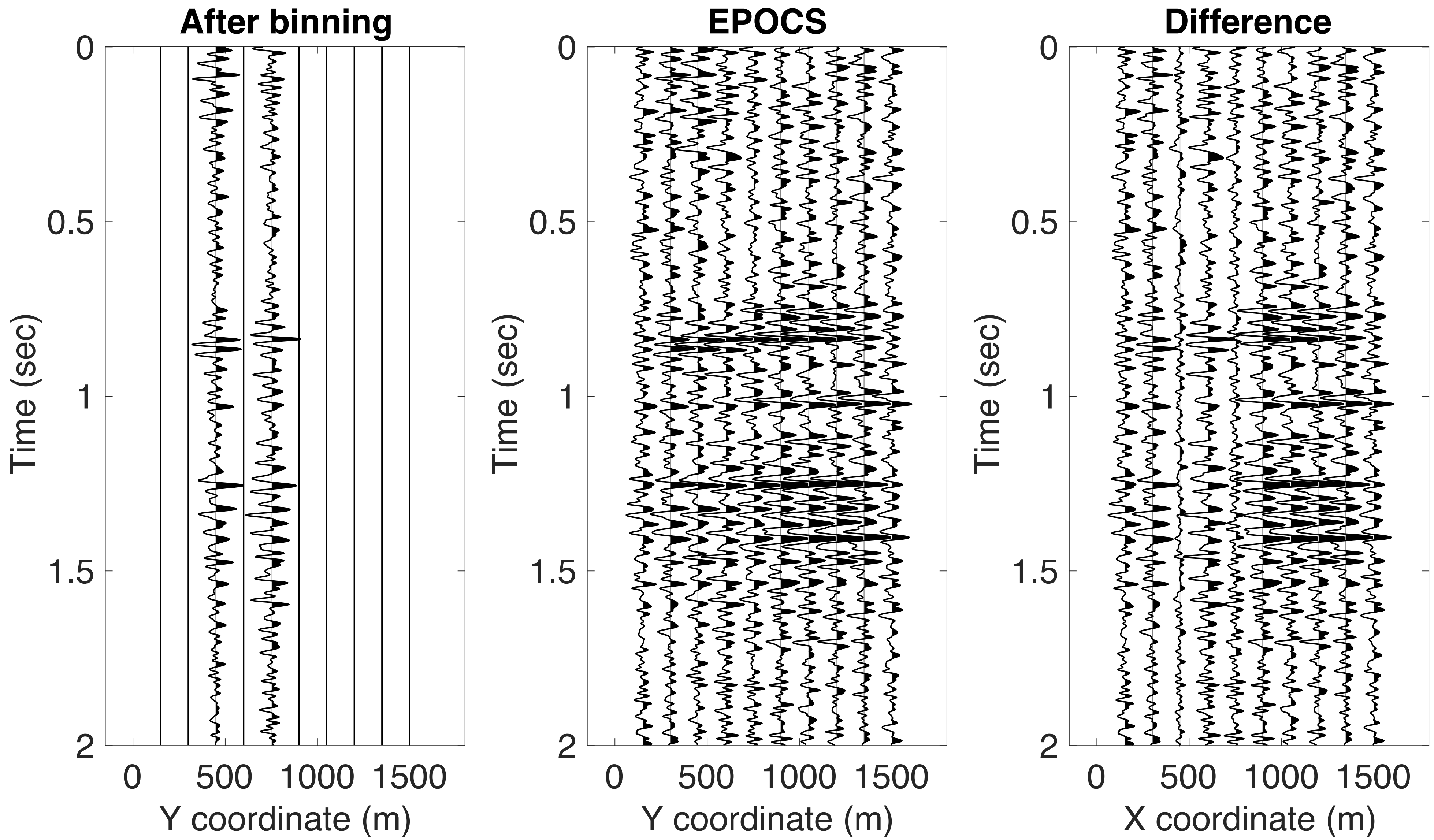
Inline slice



Real Example

- EPOCS $\mathbf{d}(:,:,17)$

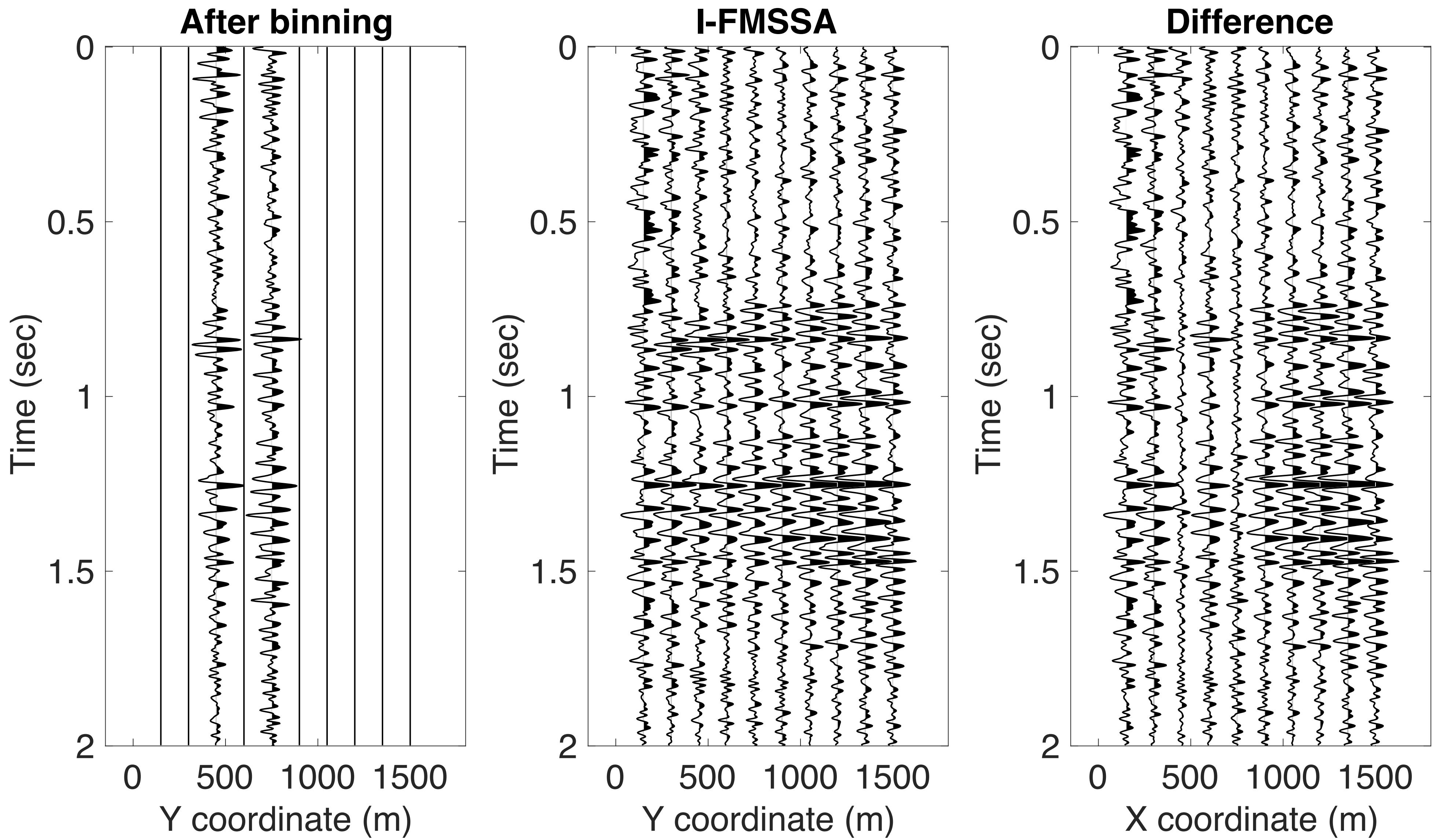
Crossline slice



Real Example

- I-FMSSA $\mathbf{d}(:,:,17)$

Crossline slice



Conclusion and Summary

- Conventional SSA methods can **be expensive due to construction of Hankel structured matrices and singular value decomposition（SVD）.**
- The applied **fast and memory efficient SSA (FMSSA)** is an appropriate substitution for MSSA with reconstruction problems.
- EPOCS method will produce “**boundary effect**” for reconstruction and is not suitable for “**noisy**” data reconstruction.
- I-FMMSA method could be a good substitution for EPOCS with application for **irregular-grid data reconstruction**, and **simultaneous denoising and reconstruction.**

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Acknowledgement

- *The sponsors of the Signal Analysis and Imaging Group (SAIG) at the University of Alberta.*