Z-99 HIGH RESOLUTION IMAGING

M.D. SACCHI[1], J. WANG[1] and H. KUEHL[2]
[1]*University of Alberta, Department of Physics, Institute for Geophysical Research*
Edmonton, AB, Canada, T6G 1W2
[2]*Shell Canada, Calgary*

Summary

Linearized inversion of seismic data entails minimizing a cost function of the form $J = ||\mathcal{L}m - d||_2^2$ where $\mathcal{L}$ denotes the forward modeling operator that maps an angle dependent reflection strength m to a measurable seismic wavefield d. Minimizing the cost function J, in the least squares sense, leads to the so-called least-squares migration methods. Migration algorithms, on the other hand, are capable of estimating a blurred version of m by using the adjoint operator $\mathcal{L}'$ (or a modified version of it). Migration methods, in general, do not attempt to fit the seismic data. Moreover, they have little control on the achievable resolution besides the one provided by the data. One way of improving resolution is by incorporating model space constraints. In this case, the cost function becomes $J = ||\mathcal{L}m - d||_2^2 + \lambda R(m)$, where R is the regularization term utilized to force the solution to exhibit desirable characteristics. We discuss the implementation of quadratic and non-quadratic constraints to generate seismic images with enhanced lateral and vertical resolution.

Introduction

Linearized inversion of seismic data requires the solution of the following problem:

$$\mathcal{L}m = d + n\,, \tag{1}$$

where d indicates the multi-source multi-receiver seismic experiment, m denotes an Earth model that consist of physical model perturbations or an angle dependent reflectivity, the operator $\mathcal{L}$ is a linearized one-way forward modeling operator computed on a known background model (macro-model), and n denotes coherent plus incoherent noise.

Rather than attempting to invert $\mathcal{L}$ via direct (analytical) methods, it has been proposed to invert m using a fitting procedure like the least squares method (Nemeth et al., 1999). What is the advantage of such a procedure? First, we can include covariance matrices in both model and data spaces, in other words the problem can be treated as a Bayesian inference problem where a priori correlations among parameters and observations can be included. Secondly, weighting matrices in data space can be used to minimize the influence of missing observations (Kuehl and Sacchi, 2003). An finally, we have the ability of obtaining figures of confidence for our final estimates of model parameters.

Quadratic and non-quadratic regularization

Regularization methods provide a procedure to guarantee the stability and uniqueness of the solution of an inverse problem. In general, we minimize a cost function of the form:

$$J = ||W_d(\mathcal{L}m - d)||_2^2 + \lambda R(m)\,. \tag{2}$$

The first term is the data misfit for a class of inference problems where we have considered Gaussian (and possible correlated) errors. In equation (2) W_d is a matrix of weights proportional to the inverse data covariance matrix. The interesting term in equation (2), R, is often called the regularization term. This term, when obtained via the Bayesian framework, is associated to the a priori distribution of parameters. Model parameters that are normally distributed and correlated lead to quadratic regularization terms of the form

$$R(m) = ||W_m\, m||_2^2 = m^T\, {W_m}^T\, W_m\, m\,. \tag{3}$$

In general, it is possible to approximate the often unknown structure of the covariance matrix by weighting matrices that control the generation of undesired features in the solution. For instance, one can penalize roughness by replacing W_m by a first or second order derivative operator.

Non-Gaussian a priori distributions of model parameters lead to non-quadratic regularization terms of the form (see for instance; Sacchi, 1997)

$$\begin{aligned} R_C(m) &= \textstyle\sum_i \ln[1+(m_i/\sigma)^2] && \text{Cauchy} \\ R_L(m) &= \textstyle\sum_i |m_i| && \text{Laplace} \end{aligned} \tag{4}$$

In the first case $R_C(m)$ is the Cauchy regularization term introduced in geophysics by Sacchi and Ulrych (1995) as a mean to achieve high resolution Radon panels for velocity analysis and de-multiple. The second selection is the l_1 criterion $R_L(m)$ that can be derived by assuming a double exponential (Laplace double exponential) distribution as a priori distribution of parameters. This class of regularization terms leads to sparse solutions. A feature that is quite often desirable when dealing with Radon transforms and problems that involve the retrieval of a parsimonious basis to represent a signal or an image. An example of the aforementioned idea is the retrieval of a finite number of harmonics from a time series immersed in white noise. In this case, imposing a sparse representation via a finite (small) number of basis functions (undamped complex exponentials) serves to reduce the spectral broadening caused by unavoidable windowing effects. It is clear that such a regularization is not the optimal one when dealing with random processes with a continuous spectral signature. These ideas are presented in Sacchi et al. (1998) with very interesting extensions provided by Giovanelli and Idier (2001) who studied the canonical problem of mixed power spectral density estimation from a finite length stationary time series.

Quadratic regularization of the migration / inversion problem

When the *inverse* of $\mathcal{L}$ represents a focusing operator that attempts to collapse certain class of events to points (i.e., Linear or Parabolic Radon Transform), R can be chosen to be a measure of sparseness, entropy or simplicity. Let us analyze the case where $\mathcal{L}$ represents a forward modeling operator that maps the distribution of physical properties to measurable seismic wavefields. In this case, a sparse solution is only valid for an Earth model that consists of a sparse distribution of isolated scatterers. This might be a good framework for certain non-invasive imaging problems in material and medical sciences but not a realistic one for seismic exploration problems. In exploration seismology, we are interested in the distribution of the reflectivity as an expression of geological boundaries. We expect certain degree of spatial continuity of reflectors with well-defined vertical and horizontal scales. All the above is complicated by the addition of structural elements like faults, folds, and unconformities. Therefore, it is clear, that a regularization term capable of expressing desirable geological features will first involve obtaining quite an important amount of information about the unknown distribution of physical properties.

A way of avoiding the aforementioned shortcoming entails parameterizing m as an angle dependent reflectivity (de Bruin et al., 1990; Prucha et al., 1999). In this case rather than having an operator $\mathcal{L}$ that maps physical properties that are independent of the seismic experiment ($[v_p, v_s, \rho]$) to measurable data d, we map *unknown data* expressed in a new domain (the angle domain) to *measurable data*. We will see that the latter simplifies the selection of the regularization term. AVA migration entails applying $\mathcal{L}'$, the adjoint of $\mathcal{L}$, to the observed data. When the data are properly sampled, the amplitude in the CIG can be corrected by incorporating the Jacobian correction Sava et al. (2001). This correction attempts to make the adjoint operator behave like the inverse operator. In general, this correction is not sufficient to achieve good amplitude fidelity. Sampling and migration artifacts are not suppressed by this correction. These artifacts can be attenuated by constraining the solution to exhibit certain degree of smoothness along the ray parameter axis. We have adopted the following cost function to retrieve a migrated image that *fi ts* the observations and, in addition, exhibits smoothness or continuity along the common image gather

$$J = ||W_d(\mathcal{L}m - d)||_2^2 + \lambda ||D_p m||^2 \tag{5}$$

The model space weights $W_m = D_p$ are given by the first order derivative operator acting along the ray parameter (Kuehl and Sacchi, 2003) or offset (Duquet, 2000). At this point it is important to mention that it is unpractical to attempt to form the normal system of equations and invert the resulting operator. We prefer to directly minimize the objective function using a conjugate gradients algorithm (Hestenes and Stiefel, 1952).

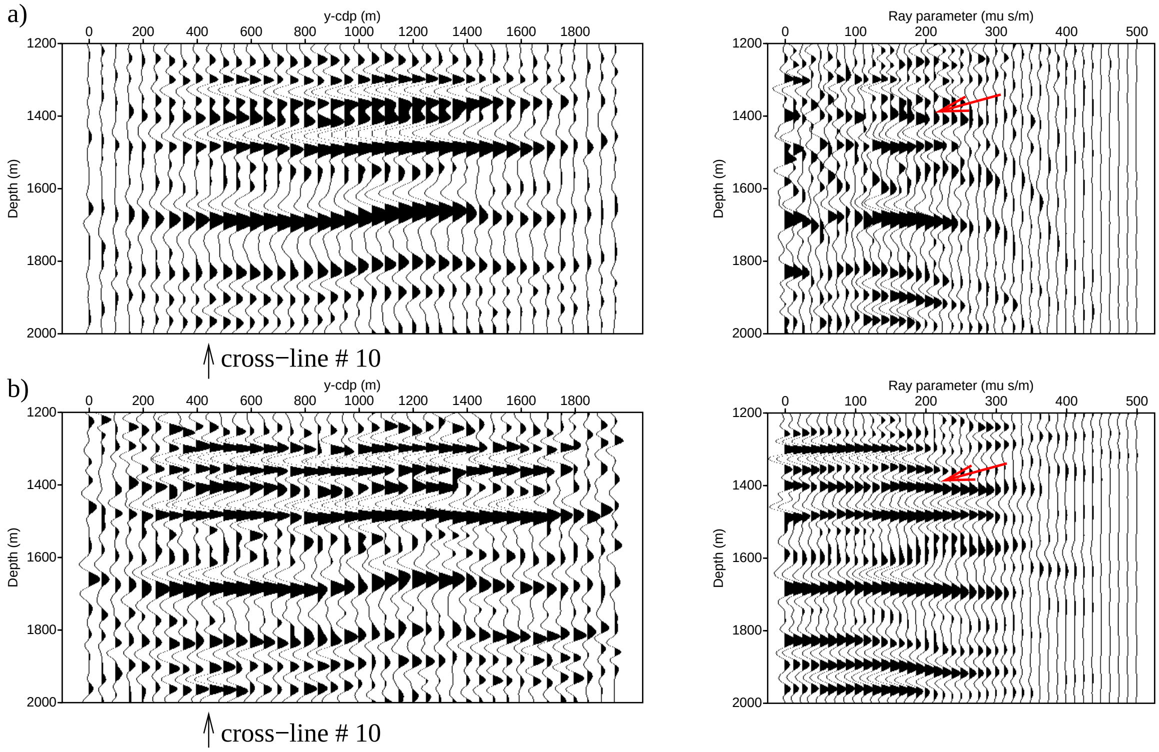


Figure 1: *a) Migrated image and associated CIG gather at cross-line position #10. b) Inversion with quadratic regularization to impose smoothness along the ray parameter axis.*

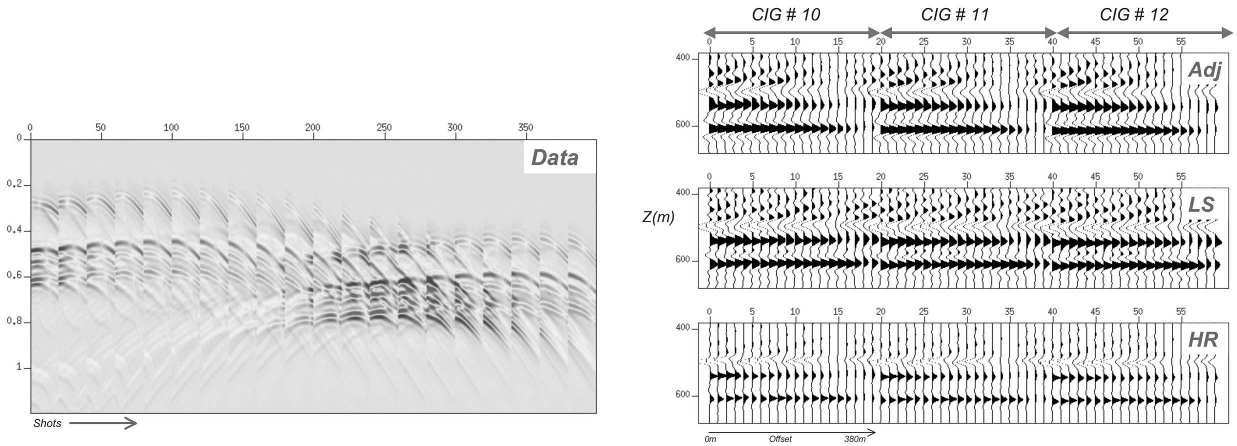
We have compared migrated versus inverted images obtained by minimizing equation (5) with a 3-D real data set from the Western Canadian Sedimentary Basin (Wang et al., 2003). The data consists of 157 in-lines and 40 cross-lines. The CMP gathers are quite sparse as the result of binning. The forward and adjoint operators are computed using a combination of common azimuth propagators (Biondi and Palacharla, 1996) and split-step correction for lateral velocity variations.

In Figure 1 we show a detailed of the migrated and inverted structural images (stacked AVP gathers) with the associated AVP gather at cross-line position # 10. It is clear that the inversion has produced a result with a considerable improvement of vertical resolution. By imposing smoothness to the inverted AVP gathers, we are able to stack individual traces in a more coherent manner. In particular, some of the smearing produced by aperture limitations (non-flatness at high ray parameters) are attenuated and, therefore, the stacked common image gather can better preserve the high frequencies. At this stage our migration schemes are able to operate with quadratic regularization strategies that can provide an important enhancement of the lateral continuity of inverted AVP gathers.

Non-quadratic regularization of the migration / inversion problem

An important enhancement of vertical resolution can be achieved by incorporating a Cauchy regularization term that forces sparsity in depth. This approach offers a bridge between well-known sparse-spike inversion methods (Oldenburg et. al, 1983) for impedance recovery and migration / inversion methods. This methodology can play an important role in the identification of thin layers and subtle stratigraphic targets; a problem often encountered in the exploration and development of new and existing plays in the Western Canadian Sedimentary Basin.

In Figure 2 we have computed 20 shot gathers in a 2D velocity model. We have estimated common image gathers via migration (adjoint operator), quadratic regularization (least squares migration with smoothing along the offset axis), and non-quadratic or high resolution imaging (smoothing along the offset axis plus sparseness in depth). Common offset Kirchoff migration and de-migration codes were used to synthesize the operators $\mathcal{L}'$ and $\mathcal{L}$, respectively. The resulting CIGs are depicted in Figure 2.



(a) (b)

Figure 2: *a) Synthetic 2D data set. b) Comparison of three migration/inversion algorithms. Top figure is the result of migrating the data in panel (a) using the adjoint operator. The central panel is the inversion with quadratic regularization used to impose horizontal smoothness in the CIG gather. The bottom panel is the high resolution solution where a Cauchy prior is used to impose sparseness in the vertical (depth) direction.*

Discussion

Imaging/inversion with the introduction of quadratic and non-quadratic constraints could lead to a new class of imaging algorithms where the resolution of the inverted image can be enhanced beyond the limits imposed by the data (band-width and aperture). This is not a completely new idea. Exploration geophysicists have been using similar concepts to invert post-stack data (sparse spike inversion) in an attempt to construct highly resolved impedance profiles. What constitutes an optimal regularization strategy for imaging problems is an open research problem. Smoothing the common image gather (along offset or ray parameter) in conjunction with a vertical sparseness constraint constitute a regularization goal that it is non-informative with respect to the structural image and consistent with the estimation of high resolution common image gathers for subsequent AVA/AVO studies.

References

Biondi, B., and Palacharla, G. 1996, 3-D prestack migration of common-azimuth data: Geophysics, **61**, no.6, 1822-1832.

de Bruin, C. G. M., Wapenaar, C. P. A., and Berkhout, A. J., 1990, Angle-dependent reflectivity by means of prestack migration: Geophysics, **55**, 1223-1234.

Duquet, B., Marfurt, K. J. and Dellinger, J.A., 2000, Kirchhoff modeling, inversion for reflectivity, and subsurface illumination: Geophysics, **65**, 1195-1209.

Hestenes, M., and Stiefel, E., 1952, Methods of conjugate gradients for solving linear systems: Nat. Bur. Standards J. Res., **49**, 403-436.

Giovanelli J. F. and Idier J., 2001, Bayesian interpretation of periodograms: IEEE Trans. on Signal Processing, 49, No. 7, 1388-1396.

Kuehl, H. and Sacchi, M.D., 2003, Least-squares wave-equation migration for AVP/AVA inversion: Geophysics, 68, 262-273.

Nemeth, T., Wu, C. and Schuster, G.T., 1999, Least-squares migration of incomplete data: Geophysics, 64, 208-221.

Oldenburg, D. W., Scheuer, T. and Levy, S., 1983, Recovery of the acoustic impedance from reflection seismograms: Geophysics, 48, 1318-1337.

Prucha, M., Biondi, B., and Symes, W., 1999, Angle-domain common image gathers by wave-equation migration: 69th Ann. Interanat. Mtg., SEG Expanded Abstracts, 824-827.

Sacchi, M.D., and Ulrych, T.J., 1995, High-resolution velocity gathers and offset space reconstruction: Geophysics, 60, 1169-1177.

Sacchi, M.D., 1997, Re-weighting strategies in seismic deconvolution: Geophys. Journal Internat., 129, 651-656.

Sacchi, M.D., Ulrych, T.J, and Walker, C., 1998, Interpolation and extrapolation using a high resolution discrete Fourier transform: IEEE Trans. on Signal Processing, 46, No. 1, 31-38.

Sava, P., Biondi, B., and Fomel, S., 2001 Amplitude-preserved common image gathers by wave-equation migration: 71th Ann. Interanat. Mtg., SEG Expanded Abstracts, 296-299.

Wang J., Kuehl H., Sacchi M.D., 2003, Least-squares wave equation AVP imaging of 3-D common azimuth data: 73rd Annual Mtg. Soc. Expl. Geophys., Expanded Abstracts, 4 pages, CDROM.