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Eliminating Blending Noise Using Fast Apex Shifted Hyperbolic Radon Transform

A. Ibrahim* (University of Alberta) & M.D. Sacchi (University of Alberta)

SUMMARY

In this work, we adopt the Stolt operator to design a robust and fast Apex Shifted Hyperbolic Radon Transform (ASHRT) that we can use to eliminate source interferences in blended sources acquisition. The problem of estimating the interference free data is posed as an inversion problem that utilizes an L1 misfit function that is not susceptible to erratic noise in the data such as blending interferences. Synthetic and real data examples show that Stolt based ASHRT can be used to eliminate interference noise efficiently.

Introduction

Recently, Ibrahim and Sacchi (2013, 2014) proposed using a robust Apex Shifted Hyperbolic Radon Transform (ASHRT) to eliminate interference noise in common receiver gathers of blended sources data. One major disadvantage of the ASHRT transform is its high computational cost. To address this limitation, Trad (2003) proposed an ASHRT that is based on Stolt migration and de-migration operators (Stolt, 1978). In this work, we adopt the Stolt operator to design a robust and fast ASHRT that we can use to eliminate source interferences in blended sources acquisition. Blended sources acquisition reduces survey cost by shortening the acquisition time and increasing subsurface illumination (Garotta, 1983; Beasley, 2008; Berkhout, 2008; Ikelle, 2010). Blended data is equivalent to time shifting individual sources data according to the sources firing times and summing them. Therefore, blended data can be generated from the single sources data by

$$\mathbf{b} = \Gamma \mathbf{D} \tag{1}$$

where $\mathbf{b}$ is the blended data, $\mathbf{D}$ represent the original data cube that would be recorded without source overlapping and Γ is the blending operator (Berkhout, 2008). Blended data $\mathbf{b}$ can be separated using the adjoint of the blending operator (*psuedodeblending* operator) as follow

$$\widetilde{\mathbf{D}} = \Gamma^T \mathbf{b}, \tag{2}$$

where $\widetilde{\mathbf{D}}$ is pseudodeblended data cube. Pseudodeblending eliminates sources delays and divide the long blended data into its equivalent non-overlapping data cube in time, source and receiver coordinates. However, pseudodeblending does not remove interferences resulting from overlapping sources and pseudodeblended data cube contains undesired interferences that can be eliminated by denoising techniques (Berkhout, 2008; Kim et al., 2009; Huo et al., 2012; Ibrahim and Sacchi, 2013, 2014). Blending interferences can be removed by denoising the data in the common receiver gather domain where this noise is incoherent (Berkhout, 2008).

Stolt operator

Stolt (1978) introduced a migration operator that map the temporal frequency ω to the vertical wavenumber k_z in Fourier domain for constant velocity using the following dispersion equation

$$\omega = \left(\frac{v}{2}\right)\sqrt{k_x^2 + k_x^2}, \tag{3}$$

where v is the migration velocity and k_x is the horizontal wavenumber. This is followed by scaling the amplitude by the factor

$$S = \frac{v}{2}\,\frac{k_z}{\sqrt{k_x^2 + k_z^2}}, \tag{4}$$

which is associated with obliquity as in Kirchhoff migration. Therefore, the adjoint Stolt (migration) operator can be written as a concatenation of three operators

$$\mathbf{L}^T = \mathbf{FFT}^{-1}_{k_z,k_x}\,\mathbf{M}^T_{\omega,k_x}\,\mathbf{FFT}_{t,x}, \tag{5}$$

and similarly the forward (de-migration) operator can be written as

$$\mathbf{L} = \mathbf{FFT}^{-1}_{\omega,k_x}\,\mathbf{M}_{k_z,k_x}\,\mathbf{FFT}_{z,x}, \tag{6}$$

where, $\mathbf{M}$ is the $f-k$ mapping operator and $\mathbf{FFT}$ is the Fast Fourier Transform operator. Although Stolt operator is derived with constant velocity assumption, it can be used to construct an ASHRT model with multiple velocities. Stolt model represents one plane inside the ASHRT model cube at constant velocity as shown in Figure 1. The classical ASHRT operator has a computational cost of $O(n_a \times n_\tau \times n_v \times n_x)$, where n_a, n_τ, n_v and n_x are the numbers of apex locations, apex times, velocities and offsets, respectively. Assuming that we scan for all possible apex locations and times, then $n_a = n_x$ and $n_\tau = n_t$. Therefore,

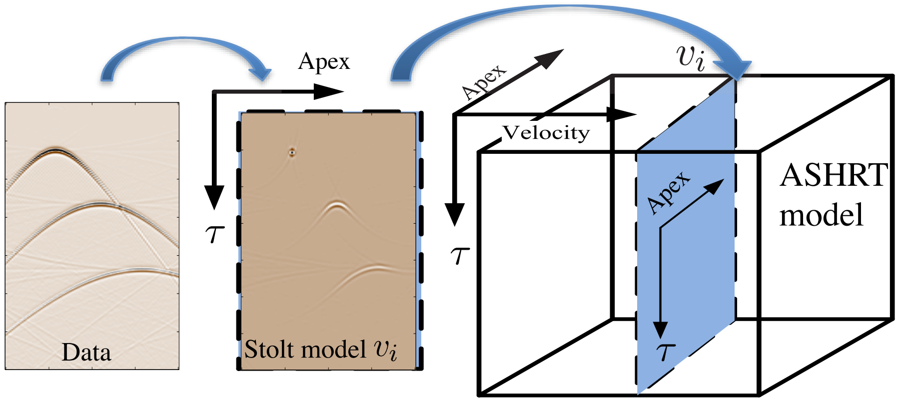

Figure 1 *Stolt operator used to compute ASHRT model.*

ASHRT operator cost is $O(n_x^2 \times n_t \times n_v)$. On the other hand, Stolt based ASHRT (without FFT zero padding) operator has a cost that is of the 2D FFT of the data with size $n_t \times n_x$ followed by $f-k$ mapping and inverse 2D FFT of the model with size $n_t \times n_v \times n_x$. Therefore, the total computational cost of an ASHRT implemented via Stlot is $O([n_t \log_2(n_t) + n_x \log_2(n_x)][n_v + 1] + n_v \times n_{kx} \times n_\omega)$, where n_{kx} and n_ω are numbers of horizontal wavenumbers and temporal frequencies, respectively. The cost of the $f-k$ mapping is proportional to $n_v \times n_{kx} \times n_\omega$ and we stress that the latter is an upper limit, since in practice we only scan for a limited band of positive frequencies and use the Fourier domain symmetry to compute the negative frequencies.

Figure 2a shows the computational times of ASHRT and Stolt operator with and without zero padding. Zero padding is sometimes required to reduce artefacts associated with $f-k$ interpolation. Figure 2b shows the improvement in the computational time of Stolt with and without zero padding compared to ASHRT. It is clear that an implementation of the ASHRT via Stolt operators can lead to a significant saving in computational costs. This is very important for processing large data set with a large number of deblended cubes.

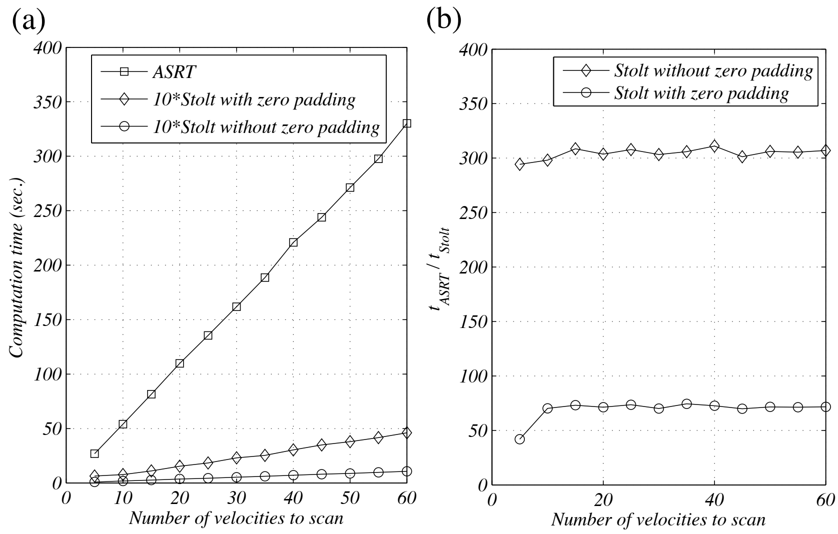

Figure 2 *Comparing operators (a) Computation times. (b) Stolt computation times compared to ASHRT.*

To estimate ASHRT model, we assume that the data is contaminated with noise and estimate the model $\mathbf{m}$ via the minimization of the cost function

$$J = \|\mathbf{d} - \mathbf{L}\mathbf{m}\|_p^p + \mu \|\mathbf{m}\|_q^q. \tag{7}$$

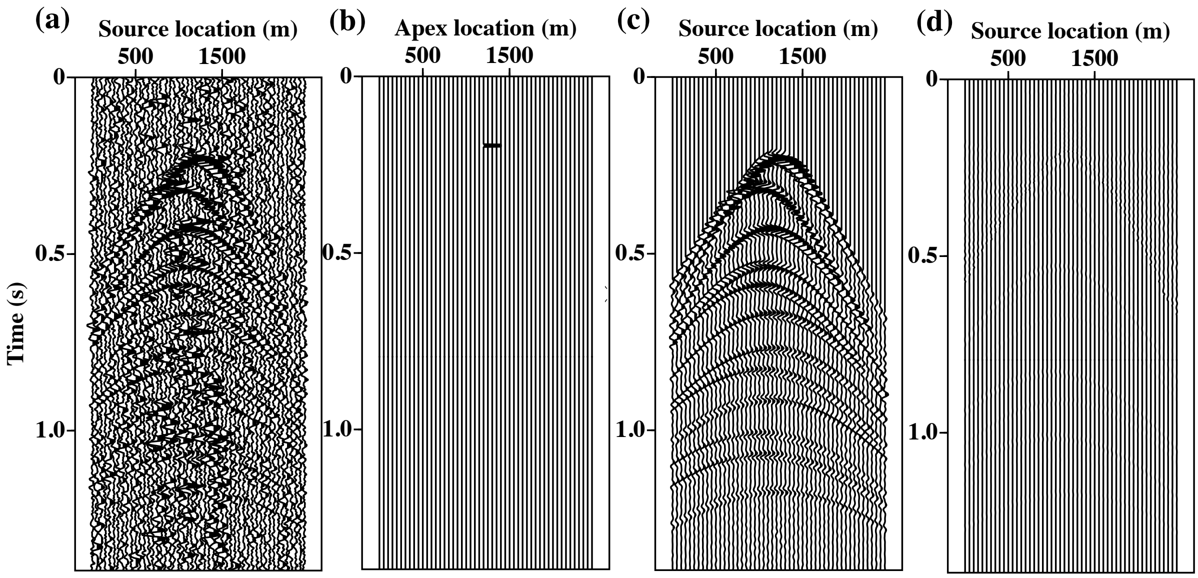

Figure 3 *Synthetic data common receiver gather. (a) Pseudodeblended gather. (a) Stolt model for one velocity estimated using $p = 1, q = 1$ inversion. (c) Data recovered by forward modelling $p = 1, q = 1$ estimated model. (d) Error in recovered data.*

In the cost function, $\mathbf{L}$ is the forward ASHRT implemented via the Stolt operator with multiple velocity panels (Trad, 2003). This is an ill-posed problem and the regularization term $\|\mathbf{m}\|_q^q$ is included to estimate a unique and stable model $\mathbf{m}$. By minimizing this cost function using Iteratively Re-weighted Least Square (IRLS) algorithm, the model of noise free data $\mathbf{m}$ is estimated. The parameters p and q represent the exponent of the p-norm of the misfit and the $q-$norm of the model regularization term, respectively. Claerbout and Muir (1973) proposed using $p = 1$ to estimate a model that is robust to erratic noise in the data such as the case of blending noise (Ibrahim and Sacchi, 2013, 2014). Since the Radon model is expected to be sparse, we can also use $q = 1$ to estimate a sparse model (Sacchi and Ulrych, 1995; Trad et al., 2003).

Examples

We tested the robust Stolt-based ASHRT with one synthetic and one marine data set from the Gulf of Mexico. Both data sets are blended numerically with a 50% time reduction compared to the conventional acquisition. The blending scheme represents one source firing with random delays. The data is pseudodeblended into common receiver gathers to obtain Figures 3a and 4a. Stolt model estimated from the pseudodeblended common receiver gather via robust inversion scheme for each data set is shown in Figures 3b and 4b. The data recovered from the robust Radon models is shown in Figures 3c and 4c. The error of the estimated data is shown in Figures 3d and 4d. The quality of the recovered data is measured using the following expression

$$Q = 10Log\frac{\|\mathbf{d}_{original}\|_2^2}{\|\mathbf{d}_{original} - \mathbf{d}_{recovered}\|_2^2}. \quad (8)$$

The Q values for the recovered synthetic data common receiver gather is 25.33 dB and for the real data common receiver gather is 11.55 dB.

Conclusions

We have implemented a fast ASHRT based on Stolt operators to eliminate blending interference noise that arises in common receiver gathers. We showed that source interferences in common receiver gathers can be removed via the ASHRT. The ASHRT presented in this article is computationally more efficient

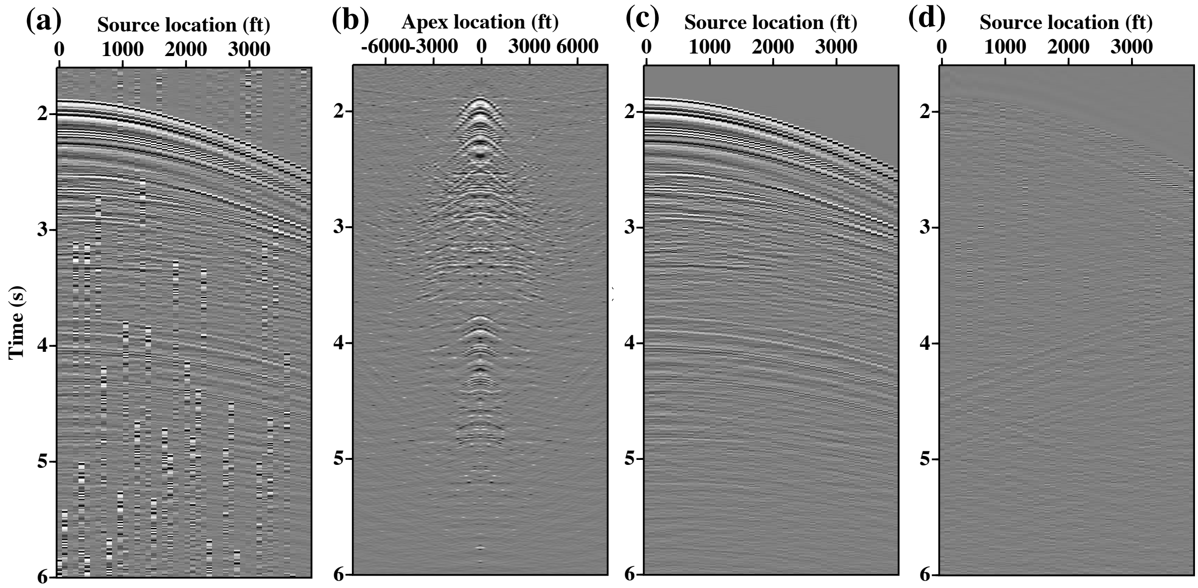

Figure 4 *Real data common receiver gather. (a) Pseudodeblended gather. (a) Stolt model for one velocity estimated using $p = 1, q = 1$ inversion. (c) Data recovered by forward modelling $p = 1, q = 1$ estimated model. (d) Error in recovered data.*

than the classical ASHRT. Last, we point out that since the Stolt operator is implemented in $f - k$ domain, it can be used in conjunction with the non-uniform FT to interpolate missing traces.

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