Least-squares split-step migration using the Hartley transform

Henning Kuehl and Mauricio D. Sacchi, University of Alberta*

Summary

Least-squares migration reduces migration artifacts and, in general, produces very accurate seismic images. Instead of mapping the seismic data into an image using a migration algorithm, the image is recovered from the data using an inversion procedure. The inversion then is implemented using the conjugate gradients method.

In this paper we propose to obtain least-squares images using the split-step migration operator and the split-step modeling operator (the adjoint of the split-step migration operator). These operators are formulated in terms of the Hartley transform. The use of the real-valued Hartley transform avoids the Hermitian symmetry inherent in the Fourier transform and allows to write algorithms that are highly optimized in both computing time and memory requirement.

In numerical simulations, we have found that the least squares image can be efficiently retrieved in a few iterations of the conjugate gradients method. The proposed algorithm can be easily implemented in parallel architecture. These features make the algorithm very attractive for inverting large data sets.

Introduction

Least-squares migration using Kirchhoff operators is an effective method to reduce migration artifacts (Nemeth et al., 1999). When the algorithm is implemented using the conjugate gradients (CG) method (Hestenes, Stiefel, 1952), the computational cost of imaging is N times the cost of applying the forward modeling operator and the migration operator, where N is the number of iterations required by the CG algorithm to achieve convergence. It is clear that to migrate seismic data in a cost efficient way, the forward and migration operators need to be optimized.

Phase-shift migration (Gazdag, 1978) is a spectral technique that is widely used in migration/modeling and is known for its computational efficiency and accuracy. The major shortcoming of phase-shift migration, however, is its restriction to media with laterally constant velocities. Phase shift plus interpolation (PSPI) migration (Gazdag, Squazzero, 1984) and split-step migration (Stoffa et al., 1990) partly overcome this limitation and can accurately handle smooth lateral variations in the velocity field.

In this presentation we favor the split-step method over the PSPI migration since split-step migration requires fewer Fourier transforms (in our case fewer Hartley transforms). Another advantage is that the adjoint, the modeling operator, can be easily coded.

To reduce the computing time and storage requirements we propose to use the real-valued Hartley transform (Bracewell, 1986) instead of the complex Fourier transform. The Hartley transform circumvents the Hermitian symmetries of the Fourier transform and leads automatically to highly optimized computer codes. The Hartley transform is suitable for implementation in parallel architectures.

Split-step migration and modeling

The use of the conjugate gradients algorithm for least-squares migration requires the forward operator and its adjoint. Interpreting migration layerwise, the split-step operator $\mathcal{L}$ can be symbolically decomposed into three linear operations $\mathcal{P}, \mathcal{C}$ and $\mathcal{S}$. With the seismic wavefield $P(x, z, \omega)$ at depth z transformed to the frequency domain the downward continuation step from z to $z + \triangle z$ consists of two linear operations. First the phase-shift term $\mathcal{P}$ is applied to the wavefield in the (k, ω) domain:

$$P_1(k, z, \omega) = P(k, z, \omega) e^{i k_{z0} \triangle z}, \tag{1}$$

where

$$k_{z_0} = \omega u_0 \sqrt{1 - \left(\frac{k}{\omega u_0}\right)^2} \tag{2}$$

and u_0 is the mean slowness for the layer interval $\triangle z$. The second step is to apply the slowness perturbation correction term $\mathcal{C}$ in the (x, ω) domain. This correction is a time shift that accounts for the lateral slowness variations $\triangle u(x) = u(x) - u_0$:

$$P(x, z + \triangle z, \omega) = P_1(x, z, \omega) e^{i\omega \triangle u(x) \triangle z}. \tag{3}$$

Finally, by summing over all frequencies (imaging principle) split-step migration can be expressed in terms of three cascaded linear operators:

$$\mathcal{L} = \mathcal{S}\mathcal{C}\mathcal{P}, \tag{4}$$

where $\mathcal{S}$ denotes the summation operator. To construct the adjoint of (4) the order of the first and last operator is interchanged and the individual adjoints are taken:

$$\mathcal{L}' = \mathcal{P}'\mathcal{C}'\mathcal{S}'. \tag{5}$$

The summation, $\mathcal{S}$, becomes a 'spraying' operation, $\mathcal{S}'$, (Claerbout, 1992). The 'spraying' operator $\mathcal{S}'$ distributes the image $I(x, z)$ at a given location (x, z) over all frequencies of $P(x, z, \omega)$. The signs of the phases in equations (1) and (3) are reversed to continue the wave field upward from the earth's interior to the surface.

The Hartley transform

Since the wave field $P(x, z, t)$ is real, its Fourier transform carries Hermitian symmetry. Hence a brute-force implementation of the split-step operators $\mathcal{L}$ and $\mathcal{L}'$ leads to inefficient computer codes. Expressed as a time-frequency transformation, the Hartley transform is defined as (Bracewell, 1986):

$$H(\omega) = \frac{1}{\sqrt{2\pi}} \int f(t) \mathrm{cas}(\omega t) dt, \qquad (6)$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int H(\omega) \mathrm{cas}(\omega t) dt, \qquad (7)$$

with the real-valued kernel $\mathrm{cas}(\omega t) = \cos(\omega t) + \sin(\omega t)$. The Hartley transform encodes the information about amplitude and phase of the wavefield in a single real function and the backward and forward transformations are of exactly the same form. For efficient coding of the Hartley transform we adopt a split-radix **F**ast **H**artley **T**ransform (FHT) that has been developed by Sorensen et al. (1985).

The phase-shift operator $\mathcal{P}$ from equation (1) in terms of the Hartley transform becomes:

$$P_1(k, z, \omega) = P(k, z, \omega) \cos(k_{z0} \triangle z) - P(-k, z, \omega) \sin(k_{z0} \triangle z). \qquad (8)$$

The slowness perturbation correction operator $\mathcal{C}$ from equation (3) is now written as

$$P(x, z + \triangle z, \omega) = P_1(x, z, \omega) \cos(\omega \triangle u(x) \triangle z) - P_1(x, z, \omega) \sin(\omega \triangle u(x) \triangle z). \qquad (9)$$

The procedure to compute the modeling migration operator $\mathcal{L}$ and its adjoint $\mathcal{L}'$ using the Hartley transform is summarized in the flowcharts in Figures 1 and 2.

Since the operators $\mathcal{L}$ and $\mathcal{L}'$ are not given as matrices, adjointness of the algorithms in Figures 1 and 2 needs to be verified. A valuable proof that one algorithm is the adjoint of the other is the dot-product test (Claerbout, 1992). The operators have to satisfy the following relation:

$$\mathbf{y}'\tilde{\mathbf{y}} = \tilde{\mathbf{x}}'\mathbf{x}, \qquad (10)$$

where $\tilde{\mathbf{y}} = \mathcal{L}\mathbf{x}$, $\tilde{\mathbf{x}} = \mathcal{L}'\mathbf{y}$. The input vectors $\mathbf{x}$ and $\mathbf{y}$ are loaded with random numbers. The algorithm presented here satisfies equation (10) down to the least significant bit.

Example

Figure 3 illustrates the image enhancement obtained by least-squares migration. To test the algorithm a band limited reflectivity model with a laterally varying velocity is generated. The forward split-step modeling operator is

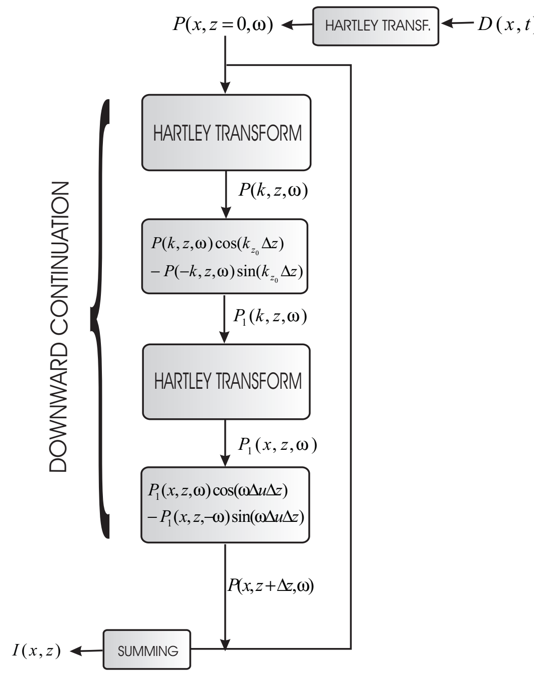

Fig. 1: The flowchart for the split-step migration using the Hartley transform. The wave field $P(x, z = 0, t) = D(x, t)$ is propagated from the surface of the earth to depth z in steps $\triangle z$. The seismic image $I(x, z)$ is constructed by applying the imaging principle .

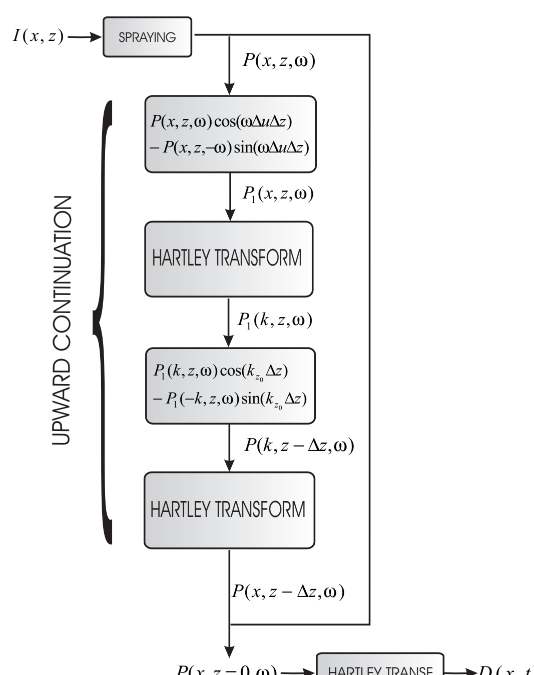

Fig. 2: The flowchart for split-step modeling using the Hartley transform. The wave field $P(x, z, t = 0) = I(x, z)$ is propagated from the interior of the earth to the surface $z = 0$ where the data $D(x, t)$ is generated.

applied to obtain the data. The standard migration using the adjoint operator yields a seismic image with strong migration artifacts and a partly incorrect reflectivity distribution. After five iterations the least-squares migration using conjugate gradients produces a high resolution image with attenuated migration artifacts. It is also important to note that the difference between the estimated reflectivity and the true reflectivity are minimal.

Conclusions

We have developed a technique to perform least-squares migration using split-step migration and its associated adjoint operator. Care has been taken in order to minimize the computational cost of the migration and the adjoint operator. In particular, we have posed our algorithms in the Hartley domain which leads to algorithms that are highly efficient in computing time and memory requirement.

The use of conjugate gradients for least-squares split-step migration improves the quality of seismic images. In general, a small number of iterations is enough to obtain a good image. A parallel implementation of the spatial Hartley transform makes the least-squares split-step migration an attractive method to process large data sets, a feature that becomes crucial in prestack migration.

Acknowledgements

This research has been partially funded by Pan Canadian Petroleum, Geo-X Ltd., and a grant from the Natural Science and Engineering Research Council of Canada.

References

Bracewell R.N., 1986: The Hartley Transform, Oxford University Press, New York.

Claerbout J.F., 1992: Earth Soundings Analysis - Processing versus Inversion, Blackwell Scientific Publications.

Gazdag J., 1978: Wave equation migration with the phase shift method, *Geophysics* 43, 1342-1351.

Gazdag J. and Squazzero P., 1984: Migration of seismic data by phase shift plus interpolation, *Geophysics* 49, 124-131.

Hestenes M.R. and Stiefel E. 1952: Methods of conjugate gradients for solving linear systems, J. Res. Natl. Bur. Stand., 49, 409-436.

Nemeth T., Wu C. and Schuster G.T., 1999: Least-squares migration of incomplete reflection data, *Geophysics* 64, 208-221.

Sorensen H.V., Jones D.L., Burrus S. and Heideman M.T., 1985: On Computing the Discrete Hartley Transform, IEEE Transactions on Acoustics, Speech and Signal Processing, Vol. ASSP-33, No.4.

Stoffa P.L., Fokkema J.T., de Luna Freire R.M. and Kessinger W.P., 1990: Split-step Fourier migration, *Geophysics* 55, 410-421.

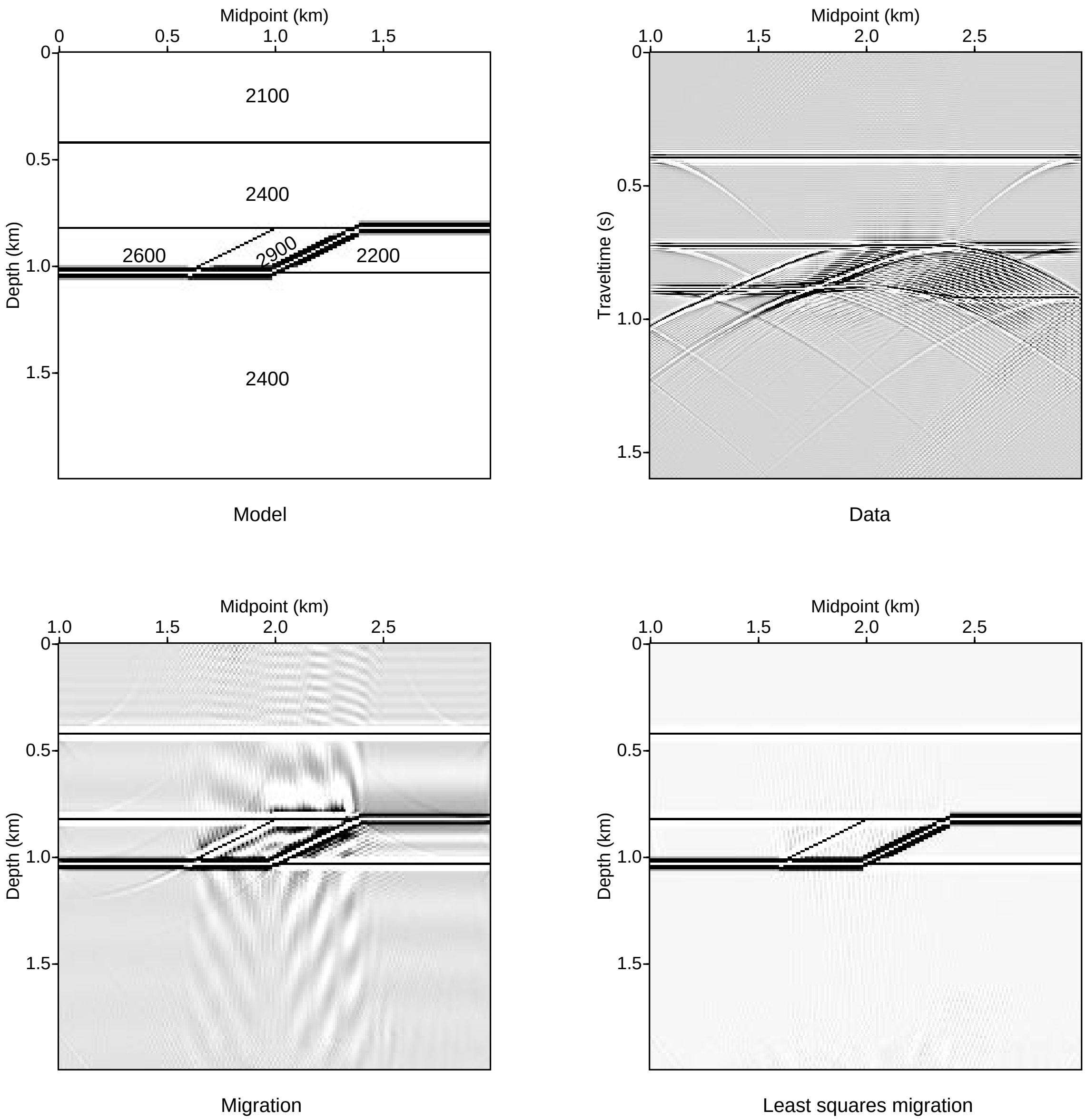

Fig. 3: A reflectivity model convolved with a Ricker wavelet is used to test the least-squares split-step migration. The interval velocities are given in meters per second. The data is generated using the forward split-step modeling operator. The split-step migrated section shows typical migration artifacts. In the least squares migrated image the artifacts are widely removed and the reflectivity is closer to that of the true model after only five iterations of the conjugate gradients scheme.