Born-WKBJ Migration/Inversion with split-step correction

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Summary

In this paper we investigate the Born-WKBJ approach to seismic modeling and migration/inversion. Following Clayton and Stolt (1981) we use the Born approximation in conjunction with the well known split-step correction (Stoffa et al., 1990) to derive a Born-WKBJ operator that is suitable for depth migration in laterally inhomogeneous media. The same theory is utilized to derive a forward or modeling operator.

It turns out that the proposed algorithms are equivalent to split-step DSR migration/modeling plus an amplitude correction term that accounts for depth-dependent background velocities.

Numerical examples using the Marmousi data set are provided.

Introduction

The Born approximation of scattering theory in conjunction with WKBJ Green functions was employed by Clayton and Stolt (1981) and Stolt and Benson (1986) to derive prestack operators for seismic modeling and inversion in the frequency-wavenumber domain. However, few numerical tests assessing this approach can be found in the literature, which is most likely due to the limitation of these operators to a stratified earth-model. Nevertheless, Born theory in the frequency wavenumber domain offers an instructive and beneficial approach to seismic migration/inversion. In this paper we compactly recast the Born-WKBJ theory in terms of forward and adjoint operators (Claerbout, 1992). It turns out that Born-WKBJ migration/inversion and modeling can be understood as conventional phase-shift migration and modeling based on the one-way wave equation plus amplitude rescaling. This amplitude-normalized downward/upward continuation is generally more consistent with the wave equation and does not present additional computational cost.

Based on the analogy between the Born scattering and the one-way wave equation we apply the well-known split-step correction (Stoffa et al., 1990) to make the Born-WKBJ operators suitable for depth migration. This results in a prestack midpoint-offset modeling and migration scheme that includes amplitude corrections for vertical variations of the background velocity field.

The concept of forward and adjoint operators lends itself automatically to least-squares migration/inversion by conjugate gradients (Nemeth et al., 1999). Kuehl and Sacchi (1999) show in numerical simulations that least-squares migration based on phase-shift techniques is able to reduce migration artifacts efficiently. However, a better understanding of the regularization of the inverse problem is necessary to make these results more useful for real-world data. The slant of scattering theory on migration/inversion and the thereof derived amplitude corrections are a step towards this goal.

In this paper we derive the Born approximation considering only velocity perturbations over the background medium. The same technique can be applied to a generalized scattering potential accounting for additional density variations (Clayton and Stolt, 1981).

Born modeling and migration

We consider the 3-D acoustic wave equation for variable velocities within a constant density medium in the frequency domain:

$$\left(\nabla^2 + \omega^2/c^2(\mathbf{x})\right)\Psi(\mathbf{x},\omega) = 0, \tag{1}$$

where the time-dependence is given by $e^{-i\omega t}$. The velocity field is split into a depth-dependent background velocity $c(z)$ and the perturbation part $\triangle c(\mathbf{x})$:

$$1/c^2(\mathbf{x}) \approx 1/c^2(z) - 2\triangle c(\mathbf{x})/c^3(z) = 1/c^2(z) + f(\mathbf{x}), \tag{2}$$

where $f(\mathbf{x})$ denotes the scattering potential in terms of the velocity inhomogeneities. The equivalent inhomogeneous wave equation is then given by

$$\left(\nabla^2 + \omega^2/c^2(z)\right)\Psi(\mathbf{x},\omega) = -\omega^2 f(\mathbf{x})\Psi(\mathbf{x},\omega). \tag{3}$$

Decomposing the total wavefield into an incident and scattered part, $\Psi_t(\mathbf{x},\omega) = \Psi_i(\mathbf{x},\omega) + \Psi(\mathbf{x},\omega)$, and using the well-known Born approximation for a weakly backscattered wavefield $\Psi(\mathbf{r},\mathbf{s},\omega)$ yields in source-receiver surface coordinates:

$$\Psi(\mathbf{r},\mathbf{s},\omega) = \omega^2 \int G(\mathbf{r},\mathbf{x},\omega) f(\mathbf{x}) G(\mathbf{x},\mathbf{s},\omega) d^3\mathbf{x}, \tag{4}$$

where the functions G are the source/receiver background Green functions for the homogeneous part of equation (3). Denoting the linear integral operator of equation (4) by $\mathcal{L}$ and using the definition of the adjoint operator $\mathcal{L}'$, $\langle \Psi_s, \mathcal{L} f\rangle = \langle \mathcal{L}'\Psi_s, f\rangle$, where the brackets represent the inner product in the (complex) data and model space, respectively, one finds the adjoint or migration operator that relates the model (the scattering potential) with the data space:

$$\tilde{f}(\mathbf{x}) = \int\!\!\int\!\!\int \omega^2 \bar{G}(\mathbf{x},\mathbf{r},\omega)\Psi(\mathbf{r},\mathbf{s},\omega)\bar{G}(\mathbf{s},\mathbf{x},\omega) d\omega d^2\mathbf{r} d^2\mathbf{s}, \tag{5}$$

where the bar denotes complex conjugation.

Miller et al. (1987) elucidate the action of the forward (modeling) and the adjoint (migration) operator in terms of integral geometry and introduce the terminology 'projection' and 'back-projection' operator, respectively. From their findings it becomes clear that $\tilde{f}(\mathbf{x})$ can be regarded as an approximation to the inverse problem. However, knowing the two operators (4) and (5) one can also invert the data in the least-squares sense $f_{LS} = (\mathcal{L}'\mathcal{L})^{-1}\mathcal{L}'\Psi_s$ to make use of the well established techniques of inverse theory.

For laterally invariant background media, however, the integrations along the surface coordinates in (4) become convolution integrals and are therefore solved in the Fourier domain (using Claerbout's (1985) sign convention throughout):

$$\begin{aligned}\Psi(\mathbf{k}_r, \mathbf{k}_s, \omega) = \omega^2 \int & G(\mathbf{k}_r, z|0, \omega) \\ & \times f(\mathbf{k}_r + \mathbf{k}_s, z) G(\mathbf{k}_s, 0|z, \omega) dz.\end{aligned} \tag{6}$$

It is convenient to transform this relation into midpoint-offset coordinates by $\mathbf{k}_r = 1/2(\mathbf{k}_y + \mathbf{k}_h)$ and $\mathbf{k}_s = 1/2(\mathbf{k}_y - \mathbf{k}_h)$,

$$\begin{aligned}\Psi(\mathbf{k}_y, \mathbf{k}_h, \omega) = \omega^2 \int & G(\mathbf{k}_y, \mathbf{k}_h, z|0, \omega) \\ & \times f(\mathbf{k}_y, z) G(\mathbf{k}_y, \mathbf{k}_h, 0|z, \omega) dz,\end{aligned} \tag{7}$$

and then use the definition of the adjoint operator again to find:

$$\begin{aligned}\tilde{f}(\mathbf{k}_y, z) = \iint & \omega^2 \bar{G}(\mathbf{k}_y, \mathbf{k}_h, z, \omega) \\ & \times \Psi(\mathbf{k}_y, \mathbf{k}_h, \omega) \bar{G}(\mathbf{k}_y, \mathbf{k}_h, z, \omega) d\omega d^2\mathbf{k}_h,\end{aligned} \tag{8}$$

which is the Born migration operator in midpoint-offset coordinates.

WKBJ Green functions

The WKBJ solution for the Fourier transformed Helmholtz equation in (3),

$$\left(d^2/dz^2 + \omega^2 q(z)^2\right) \Phi(k_x, k_y, z, \omega) = 0, \tag{9}$$

where $\omega^2 q(z)^2 = k(z)^2 = \omega^2/c(z)^2(1 - \frac{\rho^2 c(z)^2}{\omega^2})$ and $\rho^2 = k_x^2 + k_y^2$ is the squared radial surface wavenumber (of either the sources or the receivers), is given for the oscillatory part by (Stolt and Benson, 1986):

$$\Phi(z) \approx 1/\sqrt{q(z)} e^{\pm i\omega \left| \int_{z'}^{z} q(z'')dz'' \right|}, \tag{10}$$

where $\pm$ yields an down- or upward traveling wave, respectively. Making the *ansatz*

$$\begin{aligned}G(z, z') = \; & N/\sqrt{q(z)} (e^{i\omega \int_{z'}^{z} q(z'')dz''} \Theta(z - z') \\ & + e^{-i\omega \int_{z}^{z'} q(z'')dz''} \Theta(z' - z)),\end{aligned} \tag{11}$$

for the Green function with the normalization factor N and the step function $\Theta(z) = \int_{-\infty}^{z} dz' \delta(z')$ one finds that $N = -1/2(i\omega)^{-1} q(z')^{-1/2}$.

The WKBJ Green function (11) is then used in the equations (7) and (8) which yields:

$$\begin{aligned}\Psi(\mathbf{k}_y, \mathbf{k}_h, \omega) = \; & \int A(\mathbf{k}_y, \mathbf{k}_h, 0|z) \\ & \times f(\mathbf{k}_y, z) e^{i \int_0^z k(z')dz'} dz,\end{aligned} \tag{12}$$

and

$$\begin{aligned}\tilde{f}(\mathbf{k}_y, z) = \; & \int\int A(\mathbf{k}_y, \mathbf{k}_h, z|0) \\ & \times \Psi(\mathbf{k}_y, \mathbf{k}_h, \omega) e^{-i \int_0^z k(z')dz'} d\omega d^2\mathbf{k}_h,\end{aligned} \tag{13}$$

with

$$\begin{aligned}k(z) = \; & \tfrac{\omega}{c(z)} \sqrt{1 - \tfrac{(\mathbf{k}_y + \mathbf{k}_h)^2 c(z)^2}{4\omega^2}} \\ & + \tfrac{\omega}{c(z)} \sqrt{1 - \tfrac{(\mathbf{k}_y - \mathbf{k}_h)^2 c(z)^2}{4\omega^2}}.\end{aligned} \tag{14}$$

The scaling factors A in (12) and (13) are velocity and wavenumber dependent amplitude-normalization factors that yield an up- and downward continuation that is more consistent with the wave equation (Stolt and Benson, 1986). The equations (12) and (13) can be interpreted as scaled 'double-square-root' (DSR) modeling and migration operators. The exponentials are simply Gazdag's phase-shift operators for a stratified earth. The integral over z in (12) corresponds to Claerbout's (1992) 'spraying' operator for phase-shift modeling with respect to ω and $\mathbf{k}_h$ and the integration over ω and $\mathbf{k}_h$ in (13) is the imaging principle for prestack data in midpoint-offset coordinates.

Here the theory is derived for a 3 D earth but can be easily altered for a y-independent 2 D earth by setting the y components of all occurring wavenumbers to zero. Also, the modification for a $2\frac{1}{2}$ D earth can be easily accomplished (see e.g. Stolt and Benson (1986)).

Split-step correction

Since Born-WKBJ theory leads to a scaled version of Gazdag's phase-shift migration, it is justified to use any of the established correction techniques that attempt to account for lateral background velocity variations in the one-way wave equation. However, it is clear that this approach does not correct for amplitude effects due to lateral velocity changes.

For computational efficiency we use the split-step approximation for the square-root operator (Stoffa et al., 1990):

$$\sqrt{\frac{\omega^2}{c(x,z)} + \frac{\partial^2}{\partial x^2}} \approx \sqrt{\frac{\omega^2}{c(z)} + \frac{\partial^2}{\partial x^2}} + \left(\frac{\omega}{c(x,z)} - \frac{\omega}{c(z)}\right). \tag{15}$$

The vertical velocity $c(z)$ is computed by laterally averaging over the individual layers. The split-step term is applied in the space domain after an inverse Fourier transform. This correction accounts for the residual lateral velocities in term of a first order approximation.

Following Popovici (1990) we apply the split-step operator in midpoint-offset coordinates as a correction to the equations (12) and (13). This approximation has proven to be fairly accurate even for complex media like the Marmousi data set and can be efficiently implemented in parallel computer architectures.

Example

In an example we illustrate the effect of the amplitude scaling factor of equation (13) on the migrated image of the Marmousi dataset. The result obtained from the standard split-step migration in midpoint-offset coordinates is shown in Figure 1. Generally, the quality of the split-step migration is, as expected, good. The application of the scaling factor in equation (13) gives a migrated section with amplitudes closer to the true reflectivity based on the Marmousi velocity model. Since we have normalized the scaling factor with respect to the velocity of the first layer, the two migrated sections agree in the upper part of the image. The amplitudes of the deeper reflectors, however, especially the hydrocarbon trap, are more pronounced. It is important to note that the images **A** and **B** of Figure 1 were generated with the same display parameters. The image **C** in Figure 1 is an amplitude clipped version of image **B** that shows structural features more clearly than the unclipped versions.

Both migrations were efficiently performed on a SGI Origin2000 using 32 processors. It is noted that parallel processing becomes crucial for the feasibility of least-squares migration/inversion.

Conclusion

We have derived and amplitude corrected Born-WKBJ split-step algorithm. In fact, this is a simple and computationally inexpensive modification of the split-step DSR algorithm proposed by Popovici (1996). The amplitude correction yields an up- and downward continuation operator that is more consistent with the wave equation (Stolt and Benson, 1986).

Numerical simulations with the Marmousi data set show that the correction helps to preserve the relative strength of the reflectors.

Acknowledgements

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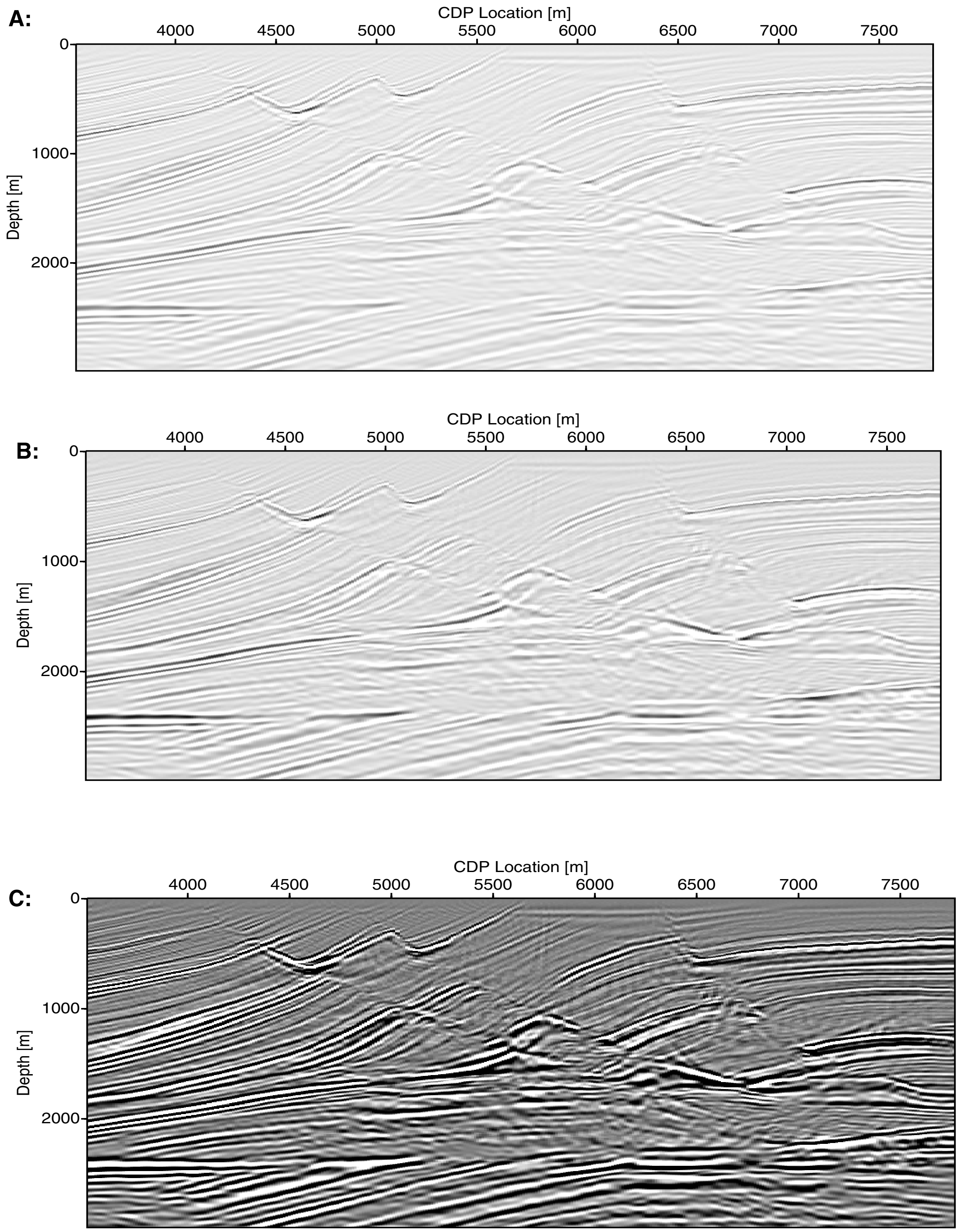

Fig. 1: **A**: Prestack split-step migration of the Marmousi dataset without amplitude scaling. **B**: Prestack split-step migration with amplitude scaling. The parameters used for plotting this image are identical to those of image **A** (no amplitude clipping). Note the stronger contrast throughout the target area. **C**: Prestack split-step migration as in image **B** but with clipped amplitudes.