Minimum weighted norm interpolation of seismic data with adaptive weights

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Summary

In seismic data processing, we often need to interpolate missing spatial locations in a domain of interest such as the shot-receiver domain. The interpolation/resampling problem can be posed as an inverse problem where from inadequate and incomplete data one attempts to recover the Fourier transform of the seismic wavefield. In this abstract, a minimum weighted norm least squares algorithm is used to invert the Fourier transform. The weighting function is designed to incorporate a priori information of the solution such as the frequency support and the shape of the spectrum.

Introduction

The interpolation/resampling problem can be posed as the inversion problem. A minimum norm least squares algorithm can be used to estimate the Fourier component of regularly sampled data from irregularly acquired data (Cary, 1997, Duijndam et al., 1999, Hindriks et al, 1997). However, the solution of the inverse problem is often ill posed due to factors such as inaccurate knowledge of data bandwidth and noise. In this case, a weighting function can be chosen to overcome the problem of non-uniqueness and instability of the solution. The criteria to choose a suitable weighting function have been discussed by several researchers (Cabrera and Thomas, 1991, Duijindam et al., 1999, Hindriks et al., 1997, Sacchi and Ulrych, 1998, Zwartjes and Duijndam, 2000).

In this abstract, we have modified the least-squares approach to include an adaptive weighted norm regularization term which incorporates a priori knowledge of energy distribution in wavenumber domain that helps to obtain a stable solution. An adaptive frequency weighted norm scheme has been proposed by Cabrera and Parks (1991) to interpolate time series. In their approaches, the missing samples are inverted rather than the coefficients of the discrete Fourier transform.

Theory and Method

We define the discrete 2-D inverse Fourier transformation in source and receiver coordinates as

$$u(x_s, x_r, \omega) = \frac{1}{MN} \sum_{n-1}^{M-1} \sum_{n=0}^{N-1} U(k_s(m) k_r(n), \omega) e^{jk_s(m)x_s} e^{jk_r(n)x_r} \tag{1}$$

where x_s and x_r are the spatial variables along source and receiver coordinates, k_s and k_r are the corresponding wavenumbers, and ω is the temporal frequency. Equation (1) gives rise to a linear system equations

$$u = AU \tag{2}$$

where

$$A_{mn} = \frac{1}{MN} e^{jk_s(m)x_s} e^{jk_r(n)x_r}, \tag{3}$$

u and U denote the known data and unknown coefficients of the DFT, respectively.

Therefore, the interpolation problem can be posed as

$$u = AU + n, \tag{4}$$

where n denotes the noise in the data. A unique solution may be obtained by minimizing the following expression:

$$J = \|AU - u\|_2 + \varepsilon \|u\|_2. \tag{5}$$

The solution of equation (5) can be shown to take the form:

$$\hat{U} = (A^H A + \varepsilon I)^{-1} A^H u, \tag{6}$$

where H denotes the transpose of a matrix.

Next, we derive a similar result but using a frequency weighting norm. In this case the function to be minimized is

$$J = \varepsilon \|u\|_Q + \|AU - u\|_2, \tag{7}$$

where

$$\| u \|_Q = \sum_k \left\{ \frac{U_k^* U_k}{Q_k} \right\} \qquad (8)$$

is a weighted DFT-domain norm (Cabrera and Parks, 1991). The weighting function Q has the same spectral support as U. Furthermore, it can be designed to incorporate the broad features of spectrum of data. The solution of equation (7) can be shown to take the form:

$$\hat{U} = (A^H A + \varepsilon Q^{-1})^{-1} A^H u. \qquad (9)$$

The elements of Q are computed from the amplitude spectrum of U. Ideally, one should know the amplitude spectrum of the data. Unfortunately, U is the unknown of our problem. The latter can be overcome by defining a iterative Q in terms of the DFT of the irregularly sampled data $A^H u$ and smoothing the result to attenuate the artifacts introduced by the irregularity of u (Ning and Nikias, 1990). The scheme can be summarized as follows:

1. Start with an initial $\hat{U}$.
2. Compute $Q = S(\hat{U}^* \hat{U})$, where S is a smoothing filter.
3. Solve $\hat{U} = (A^H A + \varepsilon Q^{-1})^{-1} A^H u$ using Conjugate Gradients.
4. Iterate until convergence.

Examples

The reconstruction is illustrated on synthetic shot gathers generated with the Marmousi model. Figure 1 shows a complete shot with 96 traces. In Figure 2, five traces are removed from the shot shown in Fig. 1. This is used as the input to test our reconstruction algorithm. The reconstruction is performed using one shot only. Figure 3 shows the reconstructed shot using a least squares method with constant weighting term (equation (6)). In Figure 4, the reconstruction is performed using the least squares algorithm with the adaptive weighting method which is introduced in this abstract.

Another example of reconstruction is demonstrated on 15 synthetic shot gathers. Figure 5 shows six of the shot gathers with the shots #3 and #7 removed. The reconstruction is performed using the adaptive weighting method. Figure 6 shows the reconstructed shot gathers (only six shots are shown). The missing shots have been completely reconstructed.

Conclusions

We have presented a 2-D interpolation algorithm that can be used to reconstruct seismic data in source-receiver coordinates.

In this method, an adaptive wavenumber domain weighting function was adopted. The algorithm is efficient and can generally give a more stable solution than the method of least squares with constant weights.

Although the methodology is illustrated with an example in shot-receiver domain, it can also be applied in the offset-mid point domain.

References

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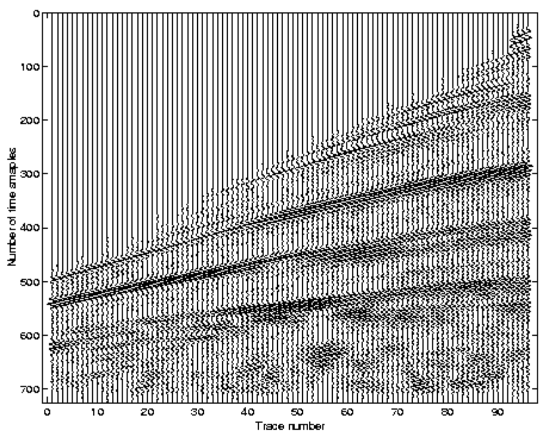

Figure 1: A sythetic complete shot gather.

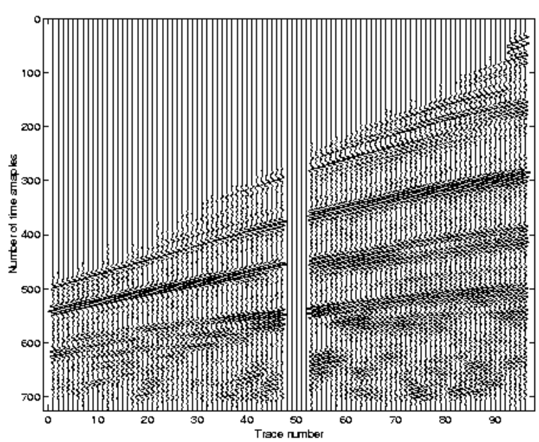

Figure 2: Sythetic shot gather with five traces removed.

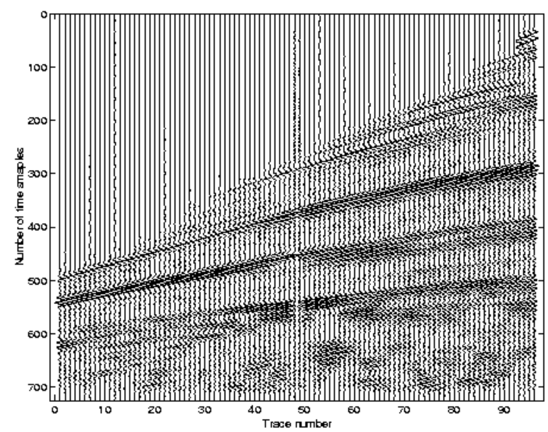

Figure 3: The reconstructed shot gather using a least squares method with constant weighting term.

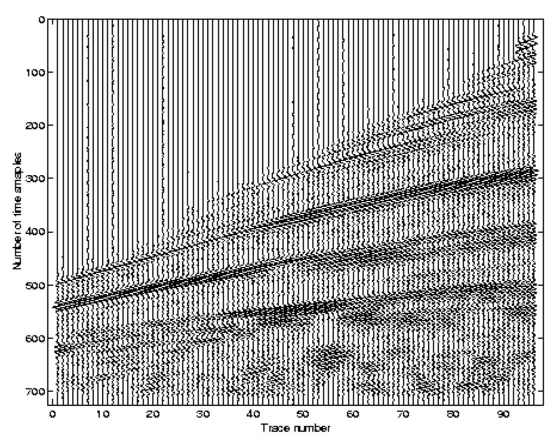

Figure 4: Reconstruction using the adaptive interpolation proposed in this paper.

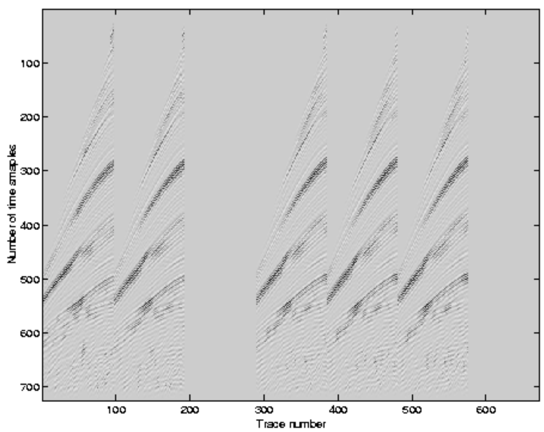

Figure 5: Six shot gathers with shots #3 and #7 removed.

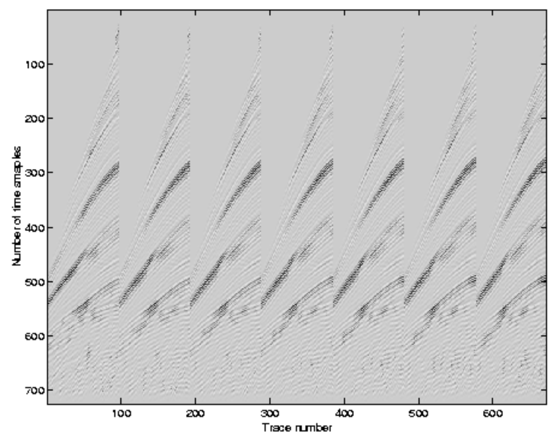

Figure 4: The reconstructed shot gathers from Fig. 5.