De-multiple via a Fast Least Squares Hyperbolic Radon Transform

Yexin Liu and Mauricio D. Sacchi, Department of Physic and Institute for Geophysical Research University of Alberta*

Summary

A fast algorithm for multiple removal based on the Hyperbolic Radon Transform (HRT) is proposed. The algorithm (Fast Hyperbolic Radon Transform (FHRT)) uses a binary classification strategy to define areas of interest in both data and model space. The main concept behind the proposed algorithm is to shrink the computational domain of the integrals (sums in the discrete case) that define the forward and adjoint Radon operators. The latter will substantially decrease the computational cost of inverting the Hyperbolic Radon operator using the method conjugate of gradients.

Synthetic and field data examples are utilized to access the feasibility of the proposed algorithm at the time of using the Hyperbolic Radon transform for de-multiple.

Introduction

The key idea in Radon based de-multiple is to map the CMP gather to a new domain where seismic reflections collapse to point-like events and, therefore, the identification and filtering of multiples become an easy task. Several authors have investigated the application of the Parabolic Radon transform to multiple removal (Hampson, 1986, Yilmaz, 1989). One advantage of the Parabolic Radon Transfom is that the Radon operator can efficiently be inverted in the frequency domain using fast solvers (Kostov, 1990, Sacchi and Porsani 1999). On the other hand, the Hyperbolic Radon transform is a time variant mapping that cannot operate in the frequency domain. Inverting the Hyperbolic Radon operator entails the inversion of a large sparse linear operator that, in general, leads to very inefficient de-multiple algorithms. The latter is an important impediment at the time of using Hyperbolic Radon de-multiple to process large data sets (Qiang, 1999).

As we have already mentioned, the Hyperbolic Radon panel is obtained by solving a large inverse problem. The latter is accomplished via the method of conjugate gradients (CG). The CG algorithm requires the specification of the forward and adjoint Hyperbolic Radon operators. It can be shown that an important part of the computational cost of the inversion hinges on the iterative application of the forward and adjoint operators. Reducing the computational cost of the aforementioned operators is the goal of this paper.

We have developed a method capable of shrinking the computational domain of the forward and adjoint Hyperbolic Radon pair. Areas of interest in both the data and the Radon panel are first identified and then, used to speed up the computational cost of the forward and adjoint Hyperbolic Radon operators. A similar method have been proposed by Li et al. (1998) and Liu and Sacchi (1999) to compress Kirchhoff migration operators. These researchers have adopted matching pursuit and wavalet transform algorithms to define “areas of interest” in data space. In this case, migration algorithms were designed to “touch” only data regions where significant energy exists.

Hyperbolic Radon Transform

The forward Hyperbolic Radon Transform is expressed by the following mapping

$$d(h,t) = \sum_{v} m(v, \tau = \sqrt{t^2 - h^2/\upsilon^2}). \qquad (1)$$

The latter can be written down in vector form as follows

$$\mathbf{d} = \mathbf{Lm}\,, \qquad (2)$$

where $d(h,t)$ indicates the CMP gather in offset-time domain and $m(v,\tau)$ the Radon panel in velocity-intercept time space. Similarly, the adjoint operator is defined as follows:

$$\tilde{m}(v,\tau) = \sum_{h} d(h, t = \sqrt{\tau^2 + (h/\upsilon)^2}) \qquad (3)$$

or

$$\tilde{\mathbf{m}} = \mathbf{L}'\mathbf{d}\,. \qquad (4)$$

Notice that since $\mathbf{L}$ is not a unitary operator $m(v,\tau) \neq \tilde{m}(v,\tau)$. It is clear that to recover $m(v,\tau)$ from $d(h,t)$ an inversion procedure for equation (1) is required. Equation (1) can be inverted using the method of conjugate gradients (CG) (see, for instance, Strang, 1986). The CG method constructs the solution that minimizes the Euclidean norm $\|\mathbf{Lm} - \mathbf{d}\|$ via a sequence of steps (iterations); in each iteration the operators $\mathbf{L}$ and $\mathbf{L}'$ are applied.

A pseudo-code for L and L'

Below we provide a very simple algorithm to implement the sums given by equations (1) and (3) (Claerbout, 1992; Trad et al., 2002):

$$
\begin{aligned}
& \textit{for}\ \ \tau = \tau_{\min} : \tau_{\max} \\
& \quad \textit{for}\ \ v = v_{\min} : v_{\max} \\
& \quad\quad \textit{for}\ \ h = h_{\min} : h_{\max} \\
& \quad\quad\quad t = \sqrt{\tau^2 + (h/v)^2} \\
& \quad\quad\quad \textit{if}\ \ \textit{adjoint};\quad m(v,\tau) = m(v,\tau) + d(h,t) \\
& \quad\quad\quad \textit{if}\ \ \textit{forward};\quad d(h,t) = d(h,t) + m(v,\tau) \\
& \quad\quad \textit{end} \\
& \quad \textit{end} \\
& \textit{end}
\end{aligned}
$$

Notice that this pseudo-code could be modified in order to take into account an interpolation operator and its adjoint. Moreover, one could also incorporate a wavelet shaping operator. The latter can become important when inverting high-resolution velocity gathers (Qiang, 1999). It is clear that computing the forward and/or adjoint operator requires a number of operations that is proportional to the size of the input domain (CMP gather or Radon Panel). One way to speed up the computation of operators (1) and (3) is by defining areas of interest in the input space. Being the input space the CMP gather when using the adjoint operator and the Radon panel (velocity gather) when using the forward operator.

By regions of interest, we understand areas where a significant amount of signal exists and therefore a non-negligible contribution to the sums given by equations (1) and (3) is expected. Once these regions have been identified, the pseudo-code for the forward operator will contain the following loops:

$$
\begin{aligned}
& \textit{for}\ \ v,\tau \in R(v,\tau(\\
& \quad \textit{for}\ \ h = h_{\min} : h_{\max} \\
& \quad ....
\end{aligned}
$$

where $R(v,\tau)$ is the region of interest associated to the input Radon panel. Similarly, a pseudo-code for the adjoint operator will contain the following loops:

$$
\begin{aligned}
& \textit{for}\ \ h,\tau \in R(h,\tau(\\
& \quad \textit{for}\ \ v = v_{\min} : v_{\max} \\
& \quad ....
\end{aligned}
$$

where, now, $R(h,\tau)$ denotes the region of interest in data space (CMP gather).

Defining regions of interest

To generate areas of interest, a fast classification procedure based on the binary image method is developed. Regions of interest for the Radon panel are found by defining first a threshold value λ_m. Then, the regions of interest are given by the pairs (v,τ) such that $|m(v,\tau)| > \lambda_m$. This analysis is carried out before applying the forward operator in each CG step. Similarly, the regions of interest for the adjoint operator are obtained by defining areas of concentration of energy at near offset traces. The latter defines the set of intercept times at which reflections occur. It is clear that this procedure will change the definition of the adjoint/forward pair from iteration to iteration, and one might think that this would affect the convergence of the CG method. Fortunately, we have found that this is only true when we attempt to use a very small threshold in an attempt to further speed up the computation of the sum given in equation (1).

Numerical Examples

We illustrate the performance of the proposed method with two examples. First, we analyze a synthetic example. Then, we examine a marine data example from the Gulf of Mexico.

Figure 1 portrays the synthetic CMP gather that we have used to test our algorithm. The synthetic CMP gather consists of 92 traces of 1751 samples per trace. Notice that the algorithm can also be applied to super-CMP gathers (ensemble of adjacent CMP gathers). We first use the least-squares Hyperbolic Transform to generate a Radon panel; the filtered Radon panel was used to generate a model of multiples. The latter was subtracted from the original data to generate the model of primaries. The de-multiple data set in Figure 2 (Left) was computed using the conventional least squares Hyperbolic Radon transform; in other words, no attempt to define regions of interest was made. Figure 2 (Right) portrays the data after de-multiple with the proposed algorithm. Table 1 reports the running times for the original Hyperbolic least-squares Radon transform (HRT) and for the proposed algorithm (Fast HRT). These results were obtained using a sequential f77 code implemented in a Linux PC running at 450 MHz.

The second example is a marine data set from the Gulf of Mexico. These data have been provided by Western Geophysical to several academic and industrial research groups to test new multiple attenuation technologies (see the special issue of The Leading Edge of January, 1999). This data set has severe multiple contamination problems. Multiple reflections are more problematic than usual despite the fact that they are weak; this is because the primaries below the salt body are weak as well. Therefore, removing the multiples without corrupting the primaries is a critical concern at the time of processing these data. Figure 3 (Left) portrays the stacked section before de-multiple. Figure 3 (Right) illustrates the stacked section after de-multiple using the fast Hyperbolic Radon transform proposed in this paper. The processing time for a single CMP gather for the HRT and fast HRT algorithms are provided in Table 1. In both cases, the same level of misfit was achieved after about 11 CG iterations. In general, we have noticed a saving of an order of 2 to 3 times with respect to the conventional least squares Hyperbolic Radon transform.

Table 1 Comparison of computing times.

Dataset	Original HRT (sec)	Fast HRT (sec)
Synthetic Data	164	52
Gulf of Mexico	113	43

Conclusions

A fast de-multiple algorithm based on the least squares Hyperbolic Radon transform was proposed. The key idea of the suggested algorithm is to define regions of interest to speed up the computation of the sums that define the forward and adjoint Radon operators.

Tests with synthetic and real data showed an important improvement with respect to the classical least-squares implementation of the Hyperbolic Radon transform using CG. It is important to stress, however, that more research needs to be done to develop Hyperbolic Radon de-multiple algorithms capable of competing in time performance with the time invariant Parabolic Radon transform.

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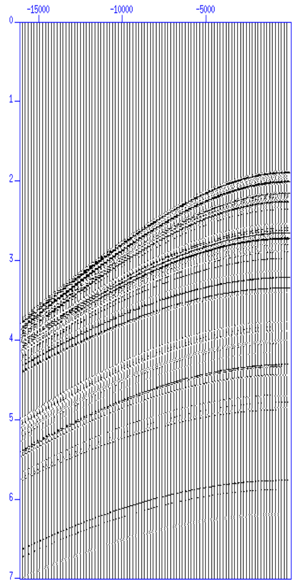

Figure 1. Synthetic CMP gather.

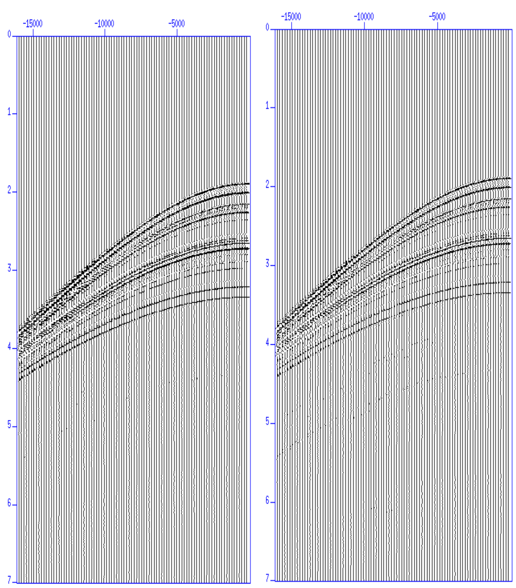

Figure 2. CMP gather after Hyperbolic Radon de-multiple. Left: least squares Hyperbolic Radon transform (HRT). Right: fast HRT.

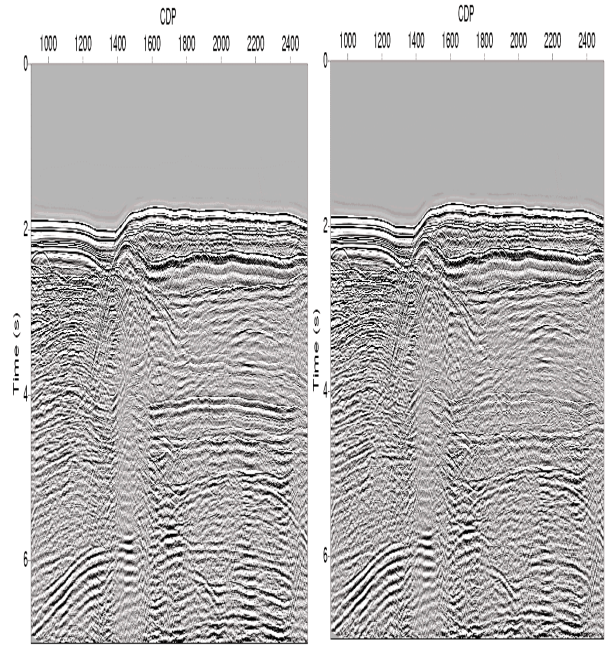

Figure 3. Left: Stacked section from the Gulf of Mexico. Right: Stacked section after fast HRT de-multiple.