Enhanced resolution in Radon domain using the shifted hyperbola equation

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Summary

The use of long-offset seismic data leads to an improvement in imaging deeper targets. The drawback is that some of the approximations commonly used in conventional processing, such as the hyperbolic moveout, do not hold for long-offset data. One of the objectives of seismic processing is to enhance the signal and remove the undesired energy from the data. A technique successfully applied in coherent noise filtering is the Radon transform. Resolution in Radon domain depends on mainly two factors: the basis function for the summation path, and the inversion algorithm. Improvements in separating signal and noise can be achieved when the appropriate basis function for the Radon transform is employed. In this paper we show that the shifted hyperbola (Castle, 1994) represents a better approximation than the normal moveout equation (Dix, 1955) for long-offset data. In particular, we focus on incorporating the shifted hyperbola formula in our current implementation of time variant Radon transforms, and developing a framework for a multiparameter Radon transform.

Introduction

Long-offset seismic data provide significant illumination for deep reflections. While it improves the imaging of deeper targets some of the approximations used in conventional processing, such as hyperbolic moveout, do not hold anymore. The main objective of seismic processing is to improve the signal-to-noise ratio. We often look to transform the data to a new domain where we can readily discriminate between signal and noise based on their different characteristics. The Radon techniques have been efficiently applied in multiple suppression, ground roll removal, and data interpolation. The Radon transform is defined as a summation along a particular path. The use of the appropriate basis function for the summation curve yields a more focused image in Radon domain making the separation of events an easier task. In this paper we study the problem of incorporating far offset approximation into the design of Radon transformations for multiple attenuation and velocity analysis. In particular, we explore the incorporation of the shifted hyperbola formula in our current implementation of time variant Radon transforms. In the current paper, we use a synthetic data example to test the viability of a multiparameter Radon transform.

Methodology

Dix (1955) introduced the normal moveout (NMO) equation for a horizontally layered-earth model

$$t = \sqrt{t_0^2 + \frac{h^2}{V_{rms}^2}} \tag{1}$$

where t is the traveltime from the geophone to the receiver, t_0 is the two-way vertical traveltime from the surface to the reflector, h is the offset, and V_{rms} is the root-mean-square velocity. This is a short offset approximation and represents the first two terms of a Taylor's series expansion of t around $h = 0$. For long offset we need to take into account at least one extra term of the expansion. Taner and Koehler (1969) give the following equation for the traveltime for a horizontally layered-earth model

$$t^2 = t_0^2 + \frac{1}{\mu_2}h^2 + \frac{1}{4}\frac{\mu_2^2 - \mu_4}{t_0^2\mu_2^4}h^4 + \frac{2\mu_4^2 - \mu_2\mu_6 - \mu_2^2\mu_4}{t_0^4\mu_2^7}h^6 + ... \tag{2}$$

Equation (2) represents an exact Taylor's series expansion of t^2 as a function of offset h. The coefficients μ_i are given by the following equation

$$\mu_i = \frac{\sum_{k=1}^{N} \Delta\tau_k V_k^i}{\sum_{k=1}^{N} \Delta\tau_k} \tag{3}$$

where $\Delta\tau_k$ is the vertical traveltime in the kth layer, and V_k is the interval velocity of the kth layer. It can be seen that $\mu_2 = V_{rms}^2$.

Two important characteristics of any equation used in seismic processing is accuracy and practicality. Although equation (2) is an accurate description of the traveltime at long offset it may not be practical to use it in data processing. Malovichko (1978, 1979) derived the shifted hyperbola NMO equation. Castle (1994) describes this equation for a horizontally layered earth as

$$t = t_0(1 - \frac{1}{S}) + \sqrt{\frac{t_0^2}{S^2} + \frac{h^2}{SV_{rms}^2}} \tag{4}$$

where

$$S = \frac{\mu_4}{\mu_2^2} \tag{5}$$

and is a dimensionless parameter called the shift parameter. From the definition of S and μ_i it can be seen that for the first layer $S = 1$ equation (4) reduces to the normal moveout equation (1) (Dix, 1955). Making use of the Jensen inequality (Claerbout, 1992) it can be proved that $S \geq 1$.

Shifted hyperbolic Radon

The Radon transform (RT) maps events with different curvatures in data domain, like primaries and multiples, to points in Radon domain. Due to this property, the Radon transform has been effectively used in coherent noise filtering, and data interpolation.

The forward Radon transform is defined as the following summation

$$d(h,t) = \sum_v m(v, \tau = \phi(v,t,h)) \tag{6}$$

where $d(h,t)$ is data in time domain, $m(v,\tau)$ represents the data in Radon domain, and $\phi(v,t,h)$ is the summation path - in this case, equation (4).

The summation can be written in matrix form as

$$\mathbf{d} = \mathbf{L}\mathbf{m} \tag{7}$$

where $\mathbf{L}$ is the Radon operator. By applying the adjoint operator $\mathbf{L}^{\mathbf{T}}$ one can obtain the data in Radon domain as

$$\hat{\mathbf{m}} = \mathbf{L}^{\mathbf{T}}\mathbf{d}. \tag{8}$$

It has been noticed by several authors (Thorson and Claerbout, 1985; Hampson, 1986; Kostov, 1990; Sacchi and Ulrych, 1995) that the utilization of equation (8) leads to a low resolution Radon panels. A key aspect in trying to circumvent the aforementioned problem entails defining the Radon transform as the solution of an inverse problem that can be solved by means of a conjugate gradient algorithm. Giving the following objective function

$$\mathbf{J} = (\mathbf{L}\mathbf{m} - \mathbf{d})^{\mathbf{T}}(\mathbf{L}\mathbf{m} - \mathbf{d}) \tag{9}$$

and minimizing it with respect to $\mathbf{m}$ we obtain the least squares (LS) solution

$$\hat{\mathbf{m}} = (\mathbf{L}^{\mathbf{T}}\mathbf{L})^{-\mathbf{1}}\mathbf{L}^{\mathbf{T}}\mathbf{d}. \tag{10}$$

Synthetic data example

Figure 2(a) shows a synthetic data shot gather that has been modeled by the shifted hyperbola equation using the parameters in Table 1. The offset is ranging from 0 to 3.5 km with a sampling interval of 0.02 km. The depth of the last reflector is 1.7 km, giving a maximum offset-to-depth ratio of approximately 2.

Figures 2(b)-(e) illustrate several shifted hyperbolic Radon panels with constant shift parameter (S) obtained using a different S for each of them. When the accurate shift parameter is used, the image in Radon domain is more focused and the correct velocity is obtained. Significant smearing of reflections occurs when a non-optimum parameter is considered and the velocity is different than the true one. When S is smaller than the true value, the obtained velocity is larger than the true velocity and frown-shaped smearing occurs. When S is too large, the obtained velocity will be too low and smile-shaped smearing occurs. In this example it has been observed that the shift parameter S is sensitive to changes of 0.05 in its value. The velocity variation with shift parameter can be better observed in Fig. 2(f) which represents a time slice at time t=1.06 s. The shift parameter varies from 1 to 1.8 with a sampling rate of 0.05. The correct values for the velocity and shift parameter at time t=1.06 s are $V_{rms} = 2.824\ km$ and $S = 1.64$. Although some kind of focusing can be observed around $S = 1.6$ it is difficult to decide what pair of V_{rms} and S values should be chosen only based on this plot.

A shifted hyperbolic Radon transform using only a constant S gives an approximate estimation of the shift parameter for different reflectors. However, the focusing in Radon domain is achieved only for the events for which S is close to the right one. A more global approach includes the use of a time dependent shift parameter. This transform would allow to scan for velocity while tuning the shift parameter. Figure 3 shows several models obtained by applying a shifted hyperbolic Radon transform with the shift parameter linearly varying with time

$$S(\tau) = S_0 + a\tau. \tag{11}$$

A priori information about S, such as the range of variation, are helpful in choosing the intercept S_0 and the slope a. As shown in de Vries and Berkhout (1984) and Sacchi et al. (1996), minimum entropy norms can be used as a measure of resolving power. Therefore, the most focused model is obtained by minimizing the entropy which is equivalent to maximizing the following function called negentropy

$$E = \frac{1}{N \ln N} \sum_{i=1}^{M} q_i \ln q_i \tag{12}$$

where

$$q_i = N \frac{m_i^2}{\sum_{k=1}^{N} m_k^2}, \tag{13}$$

N is the size of the model, and m_i represents Radon panel for a particular $S(\tau)$ curve. The maximum value of negentropy is equivalent to the maximum focusing in Radon domain, as shown in Figure 3(f) and 3(b). Figure 1 illustrates the maximum negentropy principle. In the first case, Fig. 1(a), it is shown a sparse model, with all samples zero except one whose amplitude is N (the number of samples). When computing the negentropy E, one will obtain $E = 1$. As the degree of sparseness in the model decreases, the negentropy decreases as well. The other extreme situation is the last case, Fig. 1(d), in which all the samples have the same amplitude equal to 1/N, giving a negentropy $E = 0$.

Conclusions

For long-offset data the shifted hyperbola represents a more accurate approximation. An extra unknown parameter called shift parameter is introduced. In this paper we modify our current implementation of the hyperbolic Radon transform to incorporate the shifted hyperbola formula (Castle, 1994). The quality of the results strongly depends on the shift parameter. A correct value of the parameter yields a more focused image in the Radon domain and the obtained velocity is close to the true velocity. We firstly estimate a range for the shift parameter S by applying the shifted hyperbolic Radon transform with constant S. Subsequently, a shifted hyperbolic Radon transform with variable $S(\tau)$ is applied. The most focused image in Radon domain is chosen as being the model with the maximum value of negentropy. In this synthetic example, simple linear $S(\tau)$ curves give good results, but more complicated curves can be later incorporated.

Acknowledgments

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Thickness (km)	Interval velocity (km/s)	S
0.150	1.500	1.00
0.100	1.800	1.03
0.200	2.200	1.11
0.150	2.350	1.12
0.125	2.500	1.13
0.075	2.800	1.15
0.043	3.200	1.18
0.050	3.500	1.24
0.040	3.750	1.29
0.020	3.850	1.31
0.080	4.000	1.40
0.100	4.300	1.50
0.110	4.500	1.57
0.150	4.800	1.64
0.172	5.000	1.67
0.100	5.200	1.69

Table 1: Modeling parameters.

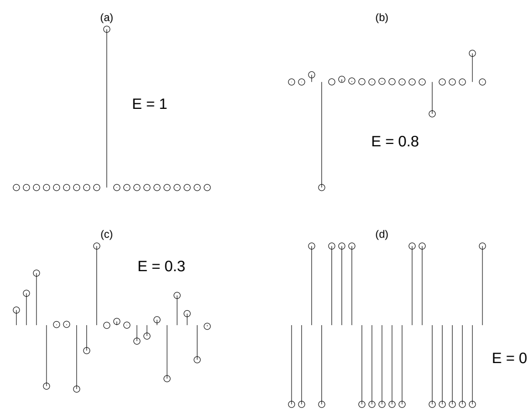

Fig. 1: Focusing measure for the shifted hyperbolic Radon transform .

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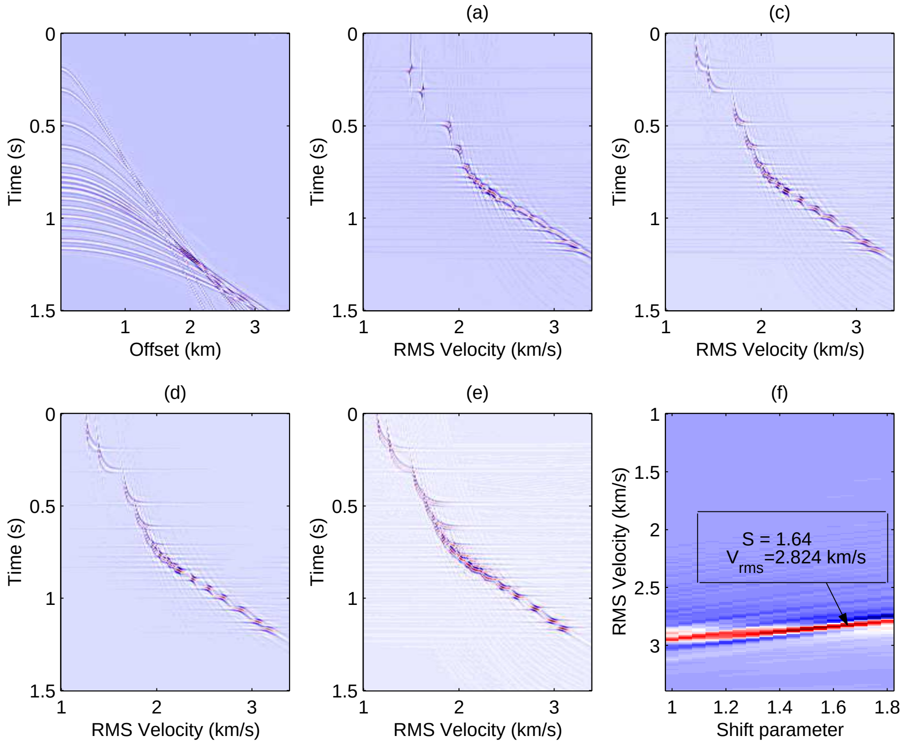

Fig. 2: Shifted hyperbolic Radon panels with constant shift parameter (S). (a) Synthetic Data. (b) Hyperbolic Radon panel (equivalent to shifted hyperbolic Radon with constant $S = 1$). (c) Shifted hyperbolic Radon panel for $S = 1.3$. (d) Shifted hyperbolic Radon panel for $S = 1.4$. (e) Shifted hyperbolic Radon panel for $S = 1.7$. (f) RMS Velocity vs. shift parameter at time t=1.06s.

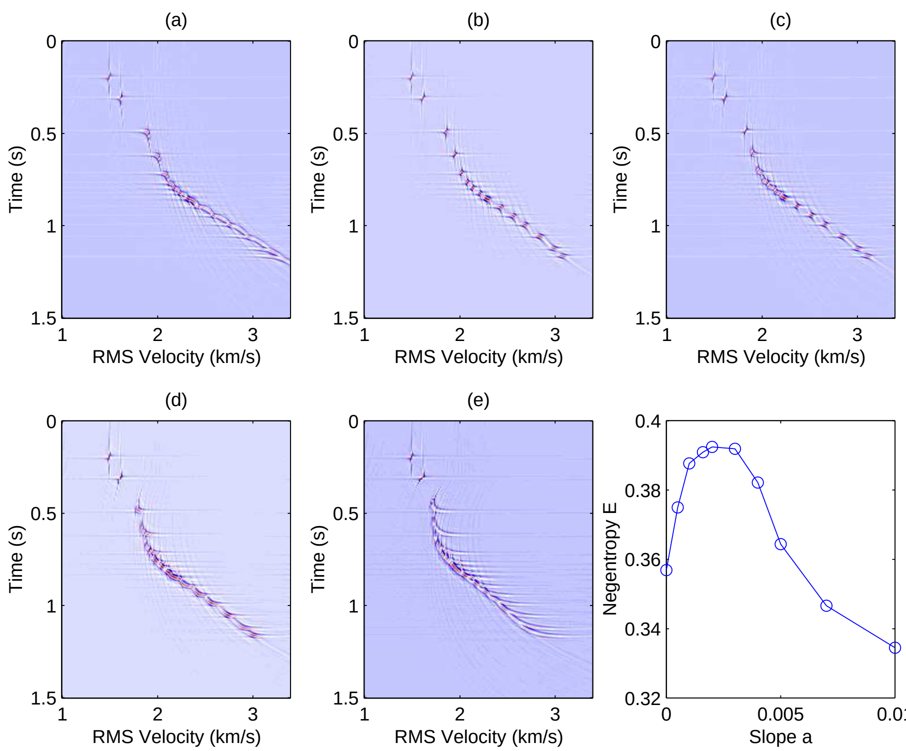

Fig. 3: Shifted hyperbolic Radon panels with variable shift parameter (S).(a) Model 1 ($S(t) = 1$). (b) Model 5 (most focused $a = 0.002$). (c) Model 6. (d) Model 8. (e) Model 10 (least focused $a = 0.01$). (f) Focusing measure curve for 10 models.