High-resolution wave equation AVP imaging with sparseness constraints

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Summary

This paper presents a new scheme for high-resolution AVP (Amplitude Variation with ray Parameter) imaging that uses non-quadratic regularization. We pose migration as an inverse problem and propose a cost function that makes use of *a* priori information about the AVP common image gather. In particular, we introduce two regularization goals: smoothness along the offset ray parameter axis and sparseness in depth. The latter yields high-resolution AVP gathers with robust estimates of amplitude variations with ray parameter. An iterative re-weighted least-squares conjugate gradients algorithm is used to minimize the cost function of the problem. We test the algorithm with synthetic data (a wedge model and the Marmousi data set). Both tests show that the method helps to improve the vertical resolution of inverted common image gathers.

Introduction

It has been shown (Nemeth et al., 1999; Duquet et al., 2000; Kuehl et al., 2002, 2003) that seismic resolution can be improved by inverting the De-migration/Migration kernel and by enforcing a regularization constraint, for example, by introducing smoothness in the solution. However, as the results of these methods show, there are many artifacts present in the solution due to operator mismatch, wave-field sampling and noise.

One possible way to further enhance the resolution and attenuate artifacts is by taking advantage of the solution itself. Iteratively using the result as a model-space regularization can lead to high-resolution artifact-free seismic images. This idea has been used in many fields of signal and image processing (Sacchi and Ulrych, 1995; Charbonnier, et al., 1997; Youzwhisen, 2001; Sacchi et al., 2003; Trad et al., 2003; Downtown and Lines, 2004). In this paper, we utilize a model-dependent sparse regularization and a model-independent smoothing regularization to estimate common image AVP gathers. Model-dependent sparse regularization is introduced via a non-quadratic norm (Cauchy norm). Smoothing, on the other hand, is implemented via a convolutional operator applied to AVP common image gathers along the ray parameter direction. This idea is used to develop an algorithm to simultaneously improve the structural interpretability and amplitude accuracy of seismic images.

It is important to point out the similarities between our algorithm and methods for impedance inversion based on sparse spike deconvolution of post-stack cubes (Oldenburg et al., 1983; Debeye and van Riel, 1990). In principle, we are using very similar concepts to find a solution that exhibits pre-defined properties such as sparseness, smoothness, etc. The main difference of our method with respect to sparse spike inversion strategies is that our operator is a one-way forward modeling operator rather than a convolutional kernel. In addition, our inversion results are in depth and the input data are prestack volumes as opposed to time-domain reflectivity estimates and post stack volumes, respectively. We believe that the proposed method provides a unifying thread between convolution-based sparse spike inversion and regularized migration/inversion methods.

Methodology

One advantage of imaging via regularized inversion is that we can make use of *a* priori information about the unknown image model (Prucha and Biondi, 2002). Robust inversion algorithms can be developed by properly honoring such information. For example, Kuehl and Sacchi (2002, 2003) showed that applying smoothing regularization in the ray parameter axis can help to remove artifacts introduced by missing information, aliasing, noise and operator mismatch. The scheme is based on the minimization of a quadratic cost function. In addition, Sacchi et al. (2003) showed that higher resolution can be acquired by solving a non-quadratic problem.

In this paper we reformulate the cost function for the least-squares wave equation AVP/AVA migration problem as follows:

$$J(\mathbf{m}) = ||\mathbf{W}(\mathbf{L}\mathbf{m} - \mathbf{d})||_2^2 + \lambda^2 F(\mathbf{S}\mathbf{D}(\mathbf{m})), \qquad (1)$$

where $\mathbf{m}$ is the earth model in terms of AVP common image gathers, $\mathbf{L}$ is a wave equation modeling operator that transforms the model to prestack seismic data, $\mathbf{d}$ is the seismic data, and $\mathbf{W}$ is a sampling matrix used to accommodate missing data in the inversion. The modeling operator is synthesized via the double-square-root upward continuation operator with split-step corrections in conjunction with a radial transform, which converts ray-parameter-dependent reflectivity to local wavefields (Kuehl and Sacchi, 2003). The operator $\mathbf{D}$ is a model-independent high-pass filter that we use to penalize non-smooth solutions, $\mathbf{S}$ is a stacking operator that converts common image gathers to a stacked image, F is a model-dependent functional used to enforce sparseness, and λ is a trade-off parameter that control the amount of regularization. By using the Cauchy norm (Sacchi and Ulrych, 1995), the sparse regularization operator F is given by

$$F(\mathbf{m}) = \Sigma_{i=1}^{n} ln(1 + m_i^2/\sigma_m^2) \qquad (2)$$

where $\sigma_{\mathbf{m}}^2$ is a scale parameter. By adopting a preconditioning strategy (Wang et al., 2004), the cost function can be expressed as follows

$$J = ||\mathbf{W}(\mathbf{LPz} - \mathbf{d})||_2^2 + \lambda^2 F(\mathbf{Sz}), \tag{3}$$

where $\mathbf{P}$ is the preconditioning matrix, and $\mathbf{z}$ is the model modified by the preconditioner. It is clear that the final solution is $\mathbf{m} = \mathbf{Pz}$. The problem can be efficiently solved by Iterative Re-weighted Least-squares (IRLS) (Scales and Smith, 1994). The cost function at the k-th iteration of the IRLS algorithm is given by

$$J(\mathbf{z}_k) = ||\mathbf{W}(\mathbf{LPz}_k - \mathbf{d})||_2^2 + \mu^2 ||\sqrt{Q_{k-1}}\mathbf{Sz}_k||_2^2, \tag{4}$$

where Q_{k-1} is a diagonal weighting matrix with diagonal elements given by

$$Q_{ii}^{k-1} = \frac{1}{1 + (m_{si}^{k-1}/\sigma_{ms}^{k-1})^2}. \tag{5}$$

In the above expression, m_{si}^{k-1} is the i-th element of the vector $\mathbf{Sz}$ at the $(k-1)$-th iteration of IRLS. Finally, σ_{ms}^{k-1} is a scale parameter, which is empirically set to some percentage of the maximum amplitude of the aforementioned vector. Application of the IRLS method involves properly choosing two hyper parameters, μ and σ_{ms}^{k-1}. The latter can be reduced to selecting one hyper-parameter, δ, by using the following expression: $\sigma_{ms}^{k-1} = \delta \cdot max(|\mathbf{m}_s|)$. Based on our experience, pairs of δ and μ with a constant product lead to similar solutions. In addition, large values of δ yield low-resolution results. Therefore in practice, we usually set δ to a small value, for example, 0.025, and adjust the other hyper parameter μ to obtain a satisfactory fitting.

The algorithm can be summarized as follows:

- We initialize $\mathbf{m} = 0$ and compute $\mathbf{Q}$
- Minimize cost function (4) via the CG algorithm.
- Update the diagonal matrix $\mathbf{Q}$, and restart the CG algorithm.

The above procedure requires about 3-4 updates (iterations) to obtain a solution that is sparse in depth and smooth with respect to the ray parameter.

Synthetic examples: a wedge model and Marmousi model

A 2-D synthetic data set was used to test the algorithm. We prepared the data by applying the forward operator $\mathbf{L}$ to a constant-velocity wedge model, which is represented by a set of AVP common image gathers (CIGs). Ideally, the inversion should be able to reconstruct the CIGs, and the stacked image of these CIGs should clearly portray the modeled structure.

We processed the data by three methods: conventional migration (the adjoint of the modeling operator), preconditioned least-squares migration (PLSM, Wang et al., 2004) and the sparse least-squares migration (SLSM) proposed in this paper. Figure 1 shows the stacked images. The result of the adjoint is quite blurry since the algorithm is not capable of reconstructing high frequencies not present in the data. On the other hand, both PLSM and SLSM are able to recover the structural images. In addition, the SLSM algorithm has produced a highly resolved CIG. This is a consequence of using a sparseness constraint that attempts to collapse the band-limiting seismic wavelet into a broad-band impulsive signal. Figure 2a-c displays a zoomed view of three common image gathers produced by these methods. The PLSM method has the ability of suppressing the sidelobes introduced by the band-limited nature of the data. To complete our analysis, we have extracted the amplitude of the tilted event and plotted AVA curves for the three methods in Figure 2d. We can observe that both PLSM and SLSM are able to preserve the amplitude response of the reflection.

We also applied the algorithm to the Marmousi data set. We randomly removed 70% of the traces to simulate a sparse data acquisition. Artifacts are present in the common image gather obtained with the migration algorithm (Figure 4a). These artifacts are substantially removed from the images obtained with PLSM and SLSM. For the purpose of comparison, we calculated the reflectivity series by using the true velocity and density model. A side-by-side comparison confirms that the SLSM has properly reconstructed the model. We observe again, as in our previous examples, an important attenuation of ringing arising from the band-limiting wavelet in the data. To evaluate the amplitude preserving properties of our algorithm, we have obtained AVA curves for the event at depth $z = 800m$. The amplitude response obtained via the migrated image is difficult to extract due to sampling artifacts. The inverted AVA responses (PLSM and SLSM), on the other hand, are in good agreement with the theoretical value.

Conclusions and discussion

We have introduced an algorithm to obtain high resolution AVA gather. The algorithm removes spurious artifacts by constraining the solution to be smooth in the ray-parameter direction and sparse in depth.

Our tests have shown that over-regularization leads to loss of valuable information that is often contained in events with small amplitudes. This problem is also encountered in techniques for post stack inversion of seismic data that are based on the sparse reflectivity assumption.

Acknowledgements

We would like to acknowledge financial support of the sponsors of the Signal Analysis and Imaging Group at the University of Alberta.

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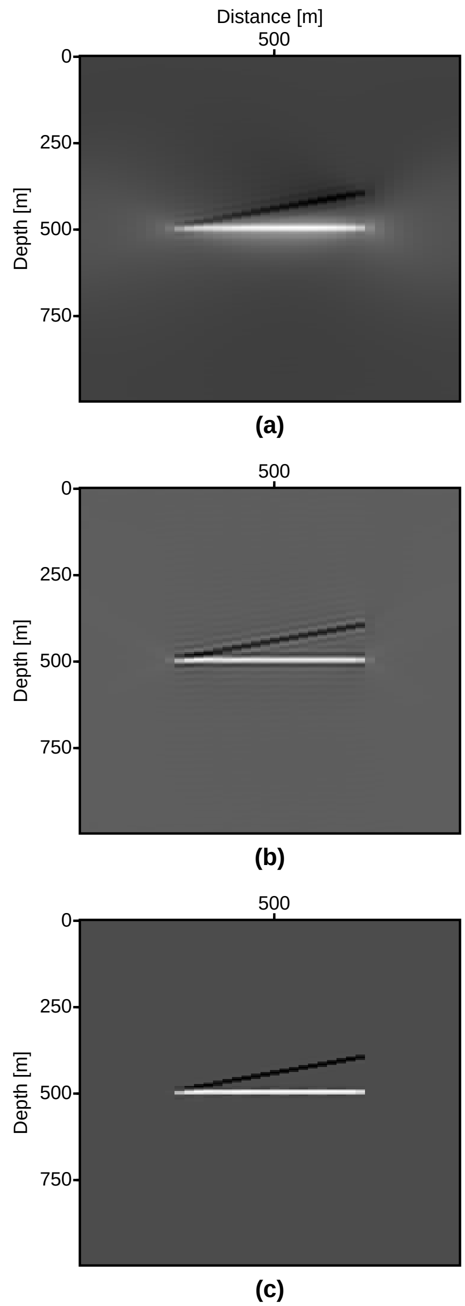

Fig. 1: Stacked images by migration, PLSM and SLSM. (a) Migration. (b) PLSM. (c) SLSM

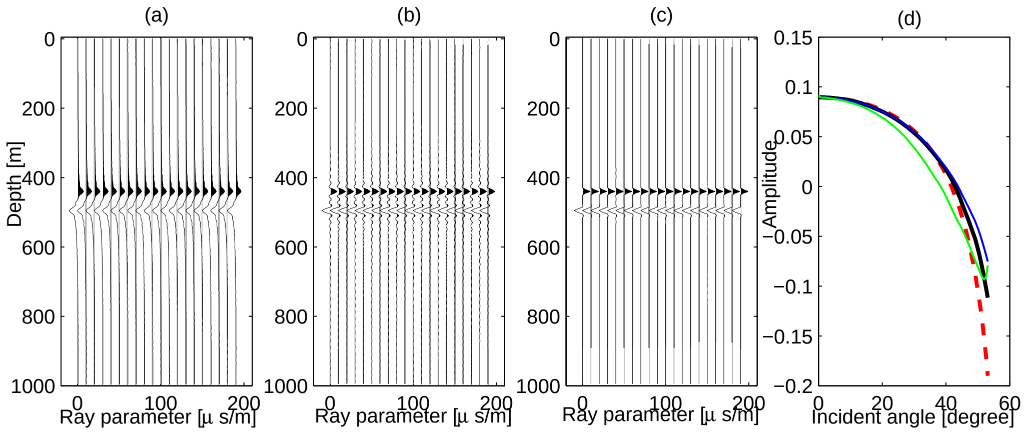

Fig. 2: Common image gathers (CIGs) and AVA curves at $x = 500m$ for the wedge model. (a) Migration. (b) Preconditioned least-squares migration (PLSM). (c) Sparse least-squares migration (SLSM). (d) AVA curves for the first event. Red dashed: the theoretical curve. Green: migration. Blue: PLSM. Black: SLSM.

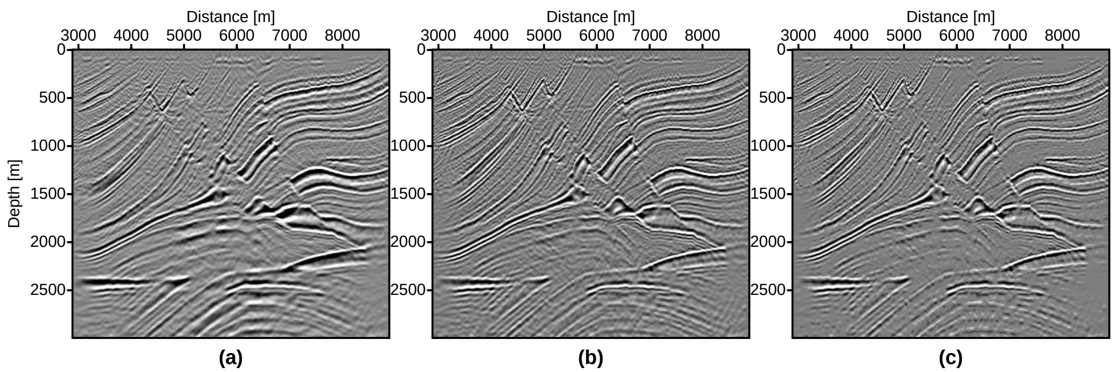

Fig. 3: Stacked images of the Marmousi data.(a) Migration. (b) PLSM. (c) SLSM.

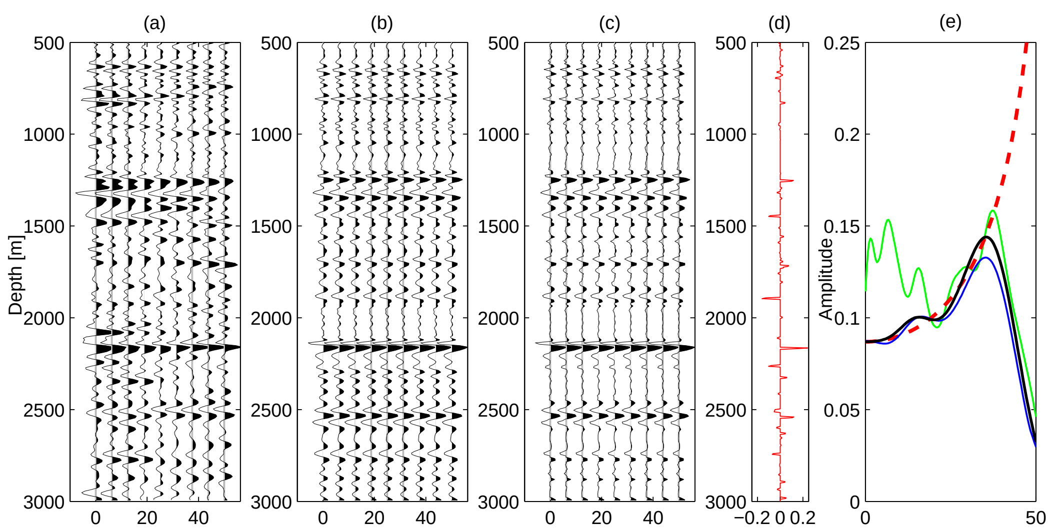

Fig. 4: Common image gathers (CIGs) and AVA curves at $x = 7500m$ for the Marmousi data. (a) Migration. (b) PLSM. (c) SLSM. (d) Zero-offset reflectivity from the density and velocity model. (e) AVA curves for the event at depth $800m$.Red dashed: the theoretical curve. Green: migration. Blue: PLSM. Black: SLSM.