Adaptive *F-X* interpolation of curved seismic events via Exponentially Weighted Recursive Least Squares (EWRLS)

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SUMMARY

The Exponentially Weighted Recursive Least Squares (EWRLS) method is adopted to estimate adaptive prediction filters for *F-X* seismic interpolation. Adaptive prediction filters are able to model signals where the dominant wave-numbers are varying in space. This concept leads to a *F-X* interpolation method that does not require windowing strategies for optimal results. Synthetic and real data examples are used to illustrate the performance of the proposed adaptive *F-X* interpolation method.

INTRODUCTION

Spitz (1991) introduced a seismic trace interpolation method that utilizes prediction filters in the frequency-space (*F-X*) domain. Spitz's algorithm is based on the fact that linear events in time-space (*T-X*) domain map to a superposition of complex sinusoids in the *F-X* domain. Complex sinusoids can be reconstructed via prediction filters (autoregressive operators); this property is used to establish a signal model for *F-X* interpolation (Spitz, 1991) and *F-X* random noise attenuation (Canales, 1984; Soubaras, 1994; Sacchi and Kuehl, 2000).

Spitz (1991) showed that prediction filters obtained at frequency f can be used to interpolate data at temporal frequency $2f$. Prediction filters estimated from the low-frequency (alias-free) portion of the data are used to interpolate the high-frequency (aliased) data components. Several modifications to Spitz's prediction filtering interpolation have been proposed. For instance, Porsani (1999) proposed a half-step prediction filter scheme that makes the interpolation process more efficient. Gulunay (2003) introduced an algorithm with similarities to *F-X* prediction filtering with a very elegant representation in the frequency-wavenumber *F-K* domain. Recently, Naghizadeh and Sacchi (2007) proposed a modification of *F-X* interpolation that allows to reconstruct data with gaps.

Seismic interpolation algorithms depend on a signal model. *F-X* interpolation methods are not an exception to the preceding statement; they assume data composed of a finite number of waveforms with constant dip. This assumption can be validated via windowing. Interpolation methods driven by, for instance, local Radon transforms (Sacchi et al., 2004) and Curvelet frames (Herrmann and Hennenfent, 2008) assume a signal model that consists of events with constant local dip. In addition, they implicitly define operators that are local without the necessity of windowing. This is an attractive property, in particular, when compared to non-local interpolation methods (operators defined on a large spatial aperture) where optimal results are only achievable when seismic events match the kinematic signature of the operator. Examples of the latter are interpolation methods based on the hyperbolic/parabolic Radon transforms (Darche, 1990; Trad et al., 2002) and migration operators (Trad, 2003).

As we have already pointed out, *F-X* methods require windowing strategies to cope with continuous changes in dominant wave-numbers (or dips in *T-X*). In this article we propose a method that avoids the necessity of spatial windows. The proposed interpolation automatically updates prediction filters as lateral variations of dip are encountered. This concepts can be implemented in a somehow cumbersome process that requires classical *F-X* interpolation in a rolling window. In this paper we have preferred to use the framework of recursive least squares (Honig and Messerschmidt, 1984; Marple, 1987) to update prediction filters in a recursive fashion. Following Spitz (1991), prediction filters estimated at temporal frequency f are used to reconstruct data at frequency $2f$. We made a fundamental modification to Spitz's method, the interpolation stage of the algorithm uses local filters obtained via adaptive estimation with EWRLS.

THEORY

Problem definition

We consider spatial data in the $F-X$ domain. The data at one monochromatic temporal frequency f are indicated by the length-N discrete signal $\underline{x} = [x_1, x_2, x_3, \dots, x_N]^T$. We assume local prediction filters of length M. The forward prediction equation is written as follows

$$x_{M+n} = p_1(n)x_{M+n-1} + p_2(n)x_{M+n-2} + \dots p_M(n)x_n + \varepsilon_{M+n} \quad (1)$$

where $\underline{p}(n) = [p_1(n), p_2(n) \dots p_M(n)]^T$ denotes the adaptive prediction filter at spatial sample n. The quantity denoted ε indicates the innovation term. The latter can be viewed as a non-stationary autoregressive model. In other words, an autoregressive model with time(space)-variant coefficients. It is important to point out that such a model can also be used to estimate evolutionary spectra for time-frequency (space-wavenumber) analysis (Priestly, 1988).

Adaptive prediction filters are estimated by minimizing the following weighted error function:

$$J(n) = \sum_{i=1}^{n} \lambda^{n-i} |x_{i+M} - \sum_{k=1}^{M} p_k(n) x_{i+M-k}|^2, \quad (2)$$

where $0 < \lambda < 1$ is the forgetting factor. This parameter is used to reduce the contribution of data samples far away from the estimation point n.

Defining the following auxiliary vector $\underline{u}(i) = [x_{i+M-1}, x_{i+M-2}, \dots, x_i]^T$ and scalar $d(i) = x_{i+M}$, the solution that minimize the error function is given by

$$\begin{aligned} \underline{p}(n) &= (\sum_{i=1}^{n} \lambda^{n-i} \underline{u}(i)\underline{u}(i)^H)^{-1} \sum_{i=1}^{n} \lambda^{n-i} \underline{u}(i) d(i) \\ &= [\Phi(n)]^{-1} \underline{\psi}(n), \end{aligned} \quad (3)$$

where

$$\Phi(n) = \sum_{i=1}^{n} \lambda^{n-i} \underline{u}(i)\underline{u}(i)^H \quad (4)$$

$$\underline{\psi}(n) = \sum_{i=1}^{n} \lambda^{n-i} \underline{u}(i) d(i). \quad (5)$$

Adaptive estimation via EWRLS

One possible solution of the adaptive prediction problem given by equation (2) involves solving (3) for each spatial position n. The latter will require the inversion of the matrix $\Phi(n)$ at spatial position n. We will circumvent the inversion of $\Phi(n)$ by using a recursive scheme where $\underline{p}(n)$ is obtained from $\underline{p}(n-1)$ and the data point $x(n)$.

The development of the recursive scheme (EWRLS) can be found in Honig and Messerschmidt (1984) and can be summarized as follows:

$$
\begin{aligned}
\text{Let } \mathbf{R}(n-1) &= \Phi^{-1}(n-1)\\
\text{Update } \underline{p} \text{ and } \mathbf{R} & \\
\underline{\omega}(n) &= \frac{\lambda^{-1}\mathbf{R}(n-1)\underline{u}(n)}{1+\lambda^{-1}\underline{u}(n)^H\mathbf{R}(n-1)\underline{u}(n)}\\
\alpha(n) &= d(n)-\underline{u}(n)^H\underline{p}(n-1) \qquad (6)\\
\underline{p}(n) &= \underline{p}(n-1)+\underline{\omega}(n)\alpha(n)\\
\mathbf{R}(n) &= \lambda^{-1}\mathbf{R}(n-1)-\lambda^{-1}\underline{\omega}(n)\underline{u}(n)^H\mathbf{R}(n-1)\,.
\end{aligned}
$$

It is evident from equation (6) that in order to initiate the recursive algorithm $\underline{p}(1)$ and $\mathbf{R}(1)$ are required. Our current implementation of the EWRLS algorithm estimates these variables from a backward (recursive) prediction model.

Interpolation using local prediction filters

In order to interpolate the data we consider spatial samples of a specific frequency f with their associated prediction filter estimated from frequency $f/2$ Spitz (1991). Consider, for instance, a prediction filter of length $M=3$, the equations for local forward and backward prediction associated to the i-th filter are given by

$$
\begin{pmatrix}
p_3(i) & p_2(i) & p_1(i) & -1 & 0 & 0 & 0\\
0 & p_3(i) & p_2(i) & p_1(i) & -1 & 0 & 0\\
0 & 0 & p_3(i) & p_2(i) & p_1(i) & -1 & 0\\
0 & 0 & 0 & p_3(i) & p_2(i) & p_1(i) & -1\\
0 & 0 & 0 & -1 & p_1^*(i) & p_2^*(i) & p_3^*(i)\\
0 & 0 & -1 & p_1^*(i) & p_2^*(i) & p_3^*(i) & 0\\
0 & -1 & p_1^*(i) & p_2^*(i) & p_3^*(i) & 0 & 0\\
-1 & p_1^*(i) & p_2^*(i) & p_3^*(i) & 0 & 0 & 0
\end{pmatrix}
\times
\begin{pmatrix}
x_i\\ x_{\frac{2i+1}{2}}\\ x_{i+1}\\ x_{\frac{2(i+1)+1}{2}}\\ x_{i+2}\\ x_{\frac{2(i+2)+1}{2}}\\ x_{i+3}
\end{pmatrix}
\approx
\begin{pmatrix}
0\\0\\0\\0\\0\\0\\0\\0
\end{pmatrix}. \qquad (7)
$$

The rational indexes indicate the desired (interpolated) samples. We first build equations similar to (7) for all possible samples i. The data samples are divided into vectors containing known samples

$$\underline{x}_k = [x_1, x_2, x_3, \ldots, x_N]^T$$

and unknown samples

$$\underline{x}_u = [x_{3/2}, x_{5/2}, x_{7/2}, \ldots, x_{(2N-1)/2}]^T$$

to finally obtain the following over-determined system of equations

$$\mathbf{A}\underline{x}_u \approx \mathbf{B}\underline{x}_k\,. \qquad (8)$$

The matrices $\mathbf{A}$ and $\mathbf{B}$ depend on the adaptive prediction filters estimated with EWRLS. The last system of equations is solved via the method of least squares

$$\underline{x}_u = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{B}\underline{x}_k\,. \qquad (9)$$

In our numerical examples we have used the method of Conjugate Gradients to solve for the unknown samples $\underline{x}_u$.

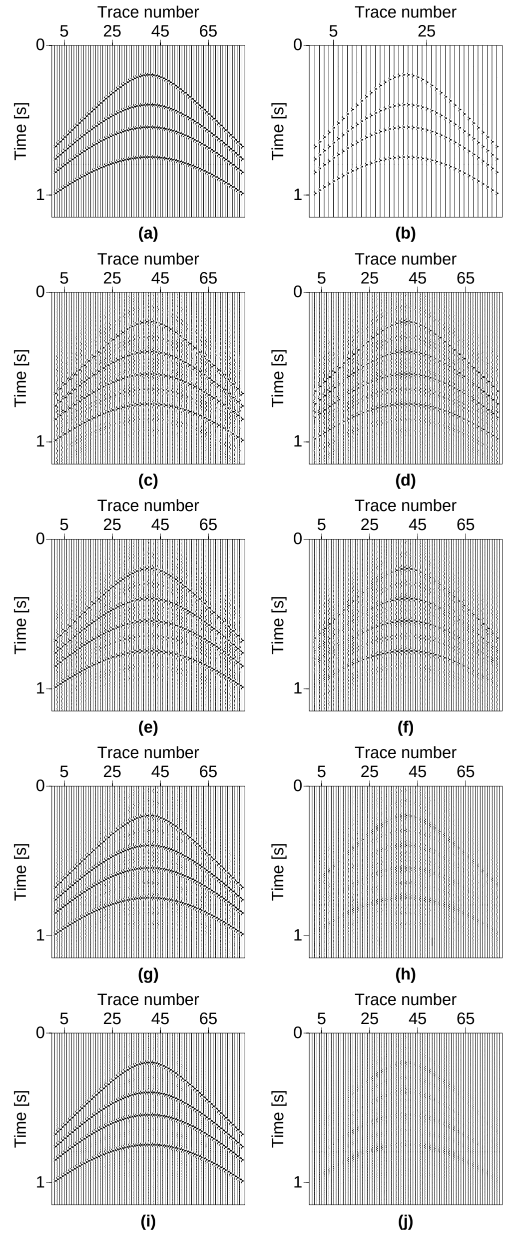

Figure 1: Comparison of Sptiz's *F-X* interpolation and adaptive *F-X* interpolation with operators computed with Exponentially Weighted Recursive Least Squares (EWRLS). a) Original data. b) Decimated data. c) Interpolation with Spitz's *FX* interpolation using the full data aperture. d) Difference between c) and a). e) Interpolated data using adaptive *F-X* interpolation with $\lambda = 1$. f) Difference between e) and a) . g) Interpolated data using windowed Spitz's *F-X* interpolation. h) Difference between g) and a). i) Interpolated data using adaptive *F-X* interpolation with $\lambda = 0.15$. j) Difference between i) and a). Length of prediction filter is $M = 4$ for all frequencies and panels.

TESTS

Synthetic example

In order to examine the performance of the adaptive *F-X* interpolation method we present two synthetic examples. In our first synthetic example we simulate a seismic gather composed of four hyperbolic events (Figure 1a). The synthetic gather is decimated (Figure 1b) and, finally, interpolated using different strategies. Figure 1c shows results obtained via *F-X* interpolation with the original algorithm proposed by Spitz. This example is quite unfair to Spitz's *F-X* interpolation because the dip of the reflection is rapidly varying with offset and windowing is required for optimal results. Figure 1e portrays results via adaptive *F-X* interpolation with forgetting factor $\lambda = 1$. The simulation with $\lambda = 1$ fails to interpolate the data. In this case, the algorithm equally weights all the observations and therefore, the prediction filters cannot adapt to changes in the local dip. As we have already mentioned, Sptiz's *F-X* method must be applied in windows to validate the assumption of constant dip waveforms. The latter is shown in Figure 1g. Figure 1i portrays the results obtained with adaptive *F-X* using forgetting factor $\lambda = 0.15$. The adaptive interpolation produces reasonable results. The algorithm heavily down-weights (forgets) the influence of samples far away form the estimation point allowing flexibility to changes in local dips. Figures 1d, 1f, 1h and 1j show error sections.

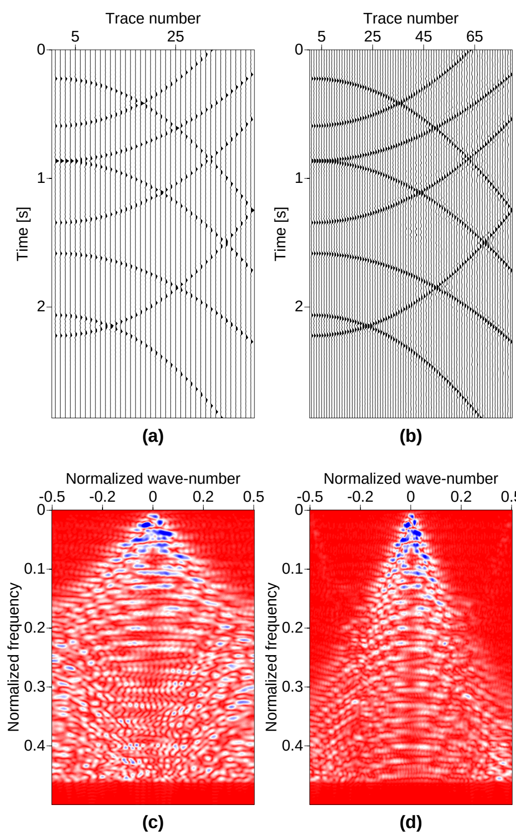

Figure 2: Synthetic example with conflicting dips. a) Original data. b) Interpolated data using adaptive *F-X* interpolation with forgetting factor $\lambda = .3$ and prediction filter length $M = 4$. c) and d) illustrate the original data and interpolated data in the *F-K* domain.

The next synthetic example consists of parabolic events with conflicting dips (Figure 2a). The original data were interpolated using adaptive *F-X* interpolation with $\lambda = 0.3$. The result is shown in Figure 2b. Figures 2c and 2d provide the *F-K* spectra of the data before and after interpolation. This example shows that adaptive *F-X* interpolation can also resolve conflicting space-variant dips. It is important to stress that an important amount of aliased energy is visible in the original data. The adaptive *F-X* interpolation has properly resolved the alias as indicated by the *F-K* panels.

Real data example

Figure 3a shows a near offset section from the Gulf of Mexico. The section was interpolated using adaptive *F-X* interpolation with $\lambda = 0.2$ and prediction filters of length $M = 4$. The final interpolation is shown in Figure 3b. It is evident that curved diffracted events were properly interpolated. Similar results were obtained using classical *F-X* interpolation with small overlapping windows of 7 traces.

PARAMETER SELECTION

For optimal results we require an automatic process for the selection of the forgetting factor λ and filter length M. We have adopted the following heuristic strategy for parameter selection. The data are first decimated. From 3 temporal frequencies we compute the average reconstruction error for different values of M and λ. The minimum reconstruction error provides optimal values M_{opt} and λ_{opt} for the decimated data. When the algorithm is used to interpolate the original data we use $M = M_{opt}$ and $\lambda = \lambda_{opt}^{1/2}$. The above-described strategy was adopted for parameter selection in the synthetic and real data examples shown in this article.

CONCLUSIONS

In this paper we introduced an efficient and easy-to-implement method to interpolate seismic records. We consider the problem of interpolating waveforms with variable dip by re-writing *F-X* interpolation as an adaptive process. The method eliminates the need of selecting window parameters (window size and amount of overlapping between adjacent windows).

The proposed adaptive *F-X* interpolation algorithm is robust under strong changes of curvature. In addition, the method performs quite well in the presence of conflicting dips with alias as illustrated by our examples. Adaptive *F-X* interpolation depends on two parameters: operator length (as in the classical *F-X* interpolation scheme) and an extra parameter, the forgetting factor, that controls adaptability to changes in local dip. We have also proposed an heuristic method to determine the operator length and forgetting factor.

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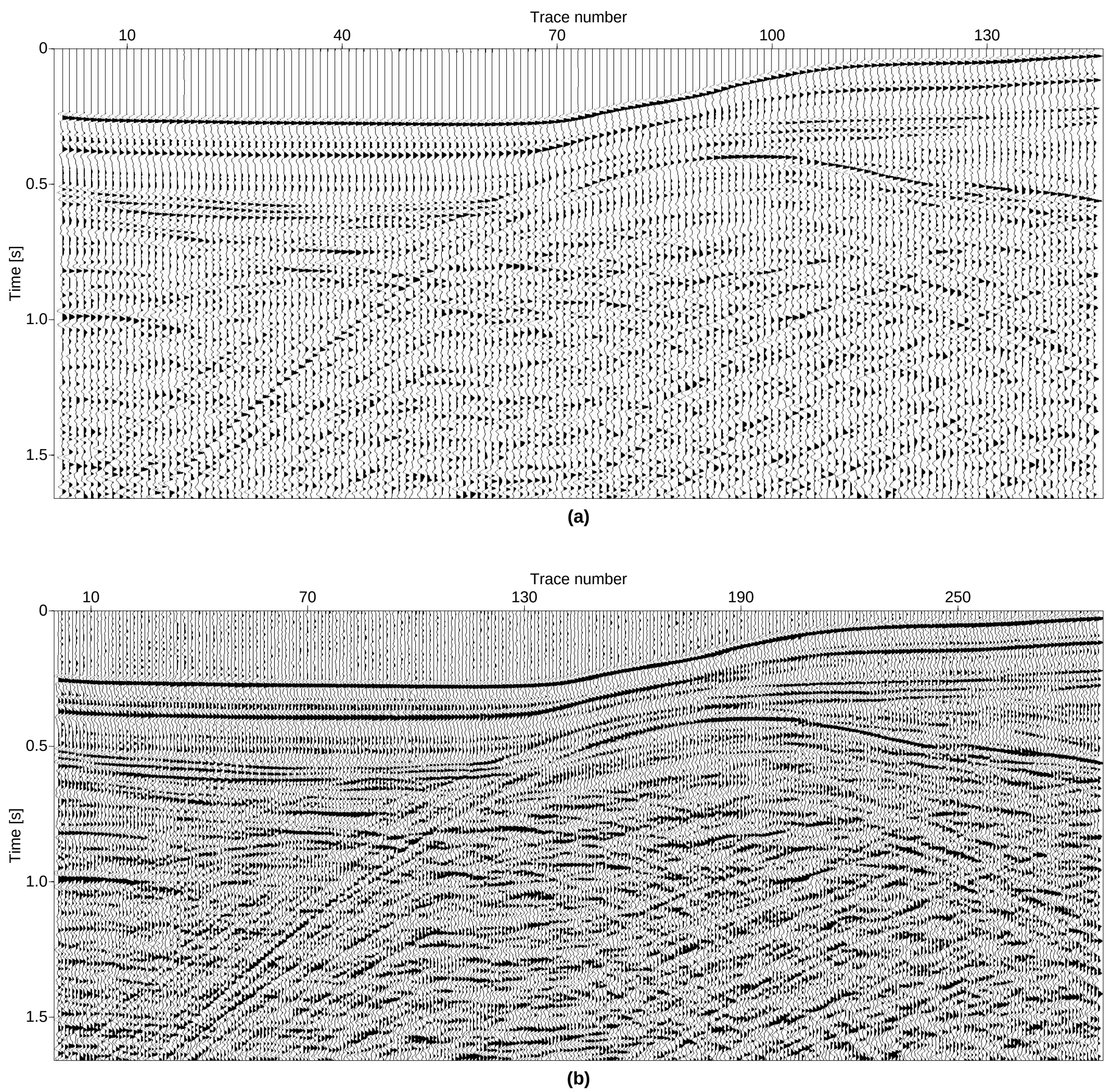

Figure 3: Real data example portraying the interpolation of a near offset section from the Gulf of Mexico. a) Original section. b) Interpolated section using adaptive *F-X* interpolation with forgetting factor $\lambda = 0.2$ and prediction filter length $M = 4$.

EDITED REFERENCES

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