Accelerating robust Radon transforms via the Stolt operator for simultaneous source separation

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SUMMARY

We use the Stolt migration/de-migration operator to eliminate simultaneous source interference noise in common receiver gathers. The Stolt operator is adopted to efficiently compute the apex shifted hyperbolic Radon model. The problem of estimating the interference free data using the Stolt operator is posed as a robust inversion problem. This inversion utilizes ℓ_1 misfit that is not susceptible to the erratic interferences in common receiver gathers. Synthetic and real data examples show that a Radon transform designed via the Stolt operator can be used to efficiently remove interferences.

INTRODUCTION

Simultaneous source acquisition permits sources to interfere and therefore, it shortens the acquisition time (Garotta, 1983; Beasley, 2008; Berkhout, 2008; Ikelle, 2010). This reduces the acquisition cost and can increase subsurface illumination by increasing source density. Moreover, Berkhout (2012) suggested using different frequency bands for sources with different spatial distribution to increase acquisition efficiency. Simultaneous source data can be synthesized from the non overlapping (conventional) sources by

$$\mathbf{b} = \Gamma\,\mathbf{D}, \tag{1}$$

where $\mathbf{b}$ is the simultaneous source data, $\mathbf{D}$ represents the non-overlapping sources data cube and Γ is the blending operator representing the sources firing times (Berkhout, 2008). The simultaneous source data $\mathbf{b}$ can be separated using the adjoint of the blending operator which is known as pseudo-deblending

$$\widetilde{\mathbf{D}} = \Gamma^T\mathbf{b}, \tag{2}$$

where $\widetilde{\mathbf{D}}$ is pseudo-deblended data cube. Pseudo-deblending eliminates the delay of sources and decompose the long blended data into its non-overlapping single source data components. However, pseudo-deblending does not remove interferences resulting from overlapping sources. These interferences are difficult to handle in processing and imaging. Therefore, simultaneous source separation (also known as deblending) is needed prior to the classical processing sequences. Simultaneous source separation methods can be broadly sorted into two main categories. In the first category, deblending is posed as direct inversion problem that minimize a cost function consisting of data misfit and a regularization term. In these methods, the cost function estimates the coefficients of the data in a transform domain. By choosing the appropriate domain, seismic data can be focused and the sparsity assumption can be used. Methods that belong to this category include sparse Radon inversion (Moore et al., 2008; Akerberg et al., 2008), iterative $f-k$ filtering (Mahdad et al., 2011; Doulgeris et al., 2012) and curvelet-based source separation (Lin and Herrmann, 2009; Wason et al., 2011). This inversion process can also be posed via a projected gradient optimization algorithm (Abma et al., 2010). A second category of simultaneous source separation methods estimate the data from the pseudo-deblended data by de-noising techniques (Beasley et al., 1998; Beasley, 2008; Kim et al., 2009; Huo et al., 2012; Ibrahim and Sacchi, 2013, 2014). These methods use the incoherent structure of interferences in gathers such as common receiver gathers (see Figure 1) to separate the sources (Berkhout, 2008). Recently,

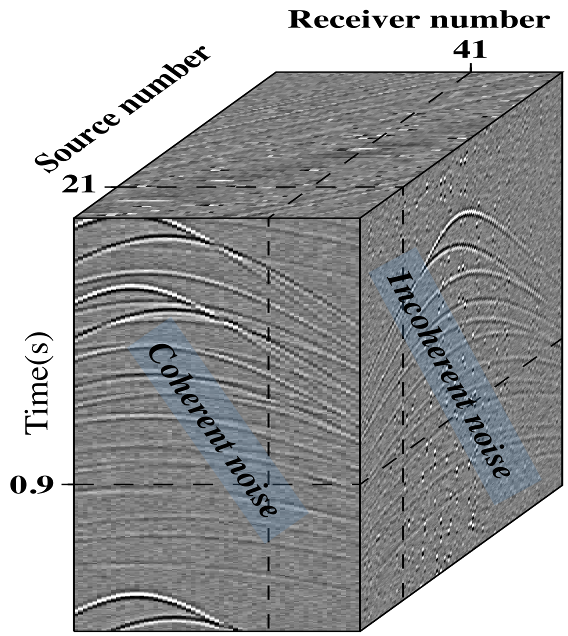

Figure 1: Pseudo-deblended data cube showing the different structure of interference noise.

Ibrahim and Sacchi (2013, 2014) used a robust Apex Shifted Hyperbolic Radon Transform (ASHRT) to eliminate interference noise in common receiver gathers of simultaneous source data. ASHRT basis match the reflection hyperbolas in common receiver gathers that makes it very suitable for denoising. However, the disadvantage of this approach is the high computational cost of the ASHRT operator. Trad (2003) proposed using the Stolt migration and de-migration operators (Stolt, 1978) to speed up the ASHRT for interpolation. In this work, we adopt the Stolt operator approach to design a robust and fast ASHRT that can be used to eliminate source interferences in common receiver gathers.

THEORY

Robust Radon Transform

In order to remove interferences by utilizing their incoherency, we need to use a suitable transform that utilizes signal coherency. Since seismic reflections can be represented by the superposition of hyperbolic events, they can be decomposed using hyperbolic Radon transforms. Transformations that use hyperbolic basis are a variant of the classical Radon transform (Radon, 1917). However, Radon transforms are non-orthogonal transformations that complicate data recovery from the model. Therefore, Thorson and Claerbout (1985) suggested casting Radon transform as an inversion problem. Since the inversion of Radon transform is an ill posed problem, a regularization (penalty) term is included in inversion cost function to estimate a stable and unique model. The general form of Radon inversion cost function is

$$J = \|\mathbf{d} - \mathbf{L}\mathbf{m}\|_p^p + \mu\|\mathbf{m}\|_q^q$$

where $\mathbf{d}$ is the data, $\mathbf{m}$ is Radon model, $\mathbf{L}$ is Radon operator and μ is the trade off parameter. In this cost function p and q represent the norms used to measure the misfit term $\|\mathbf{d} - \mathbf{L}\mathbf{m}\|$ and the model regularization term $\|\mathbf{m}\|$, respectively. One popular model regularization is the ℓ_2 norm (Beylkin, 1987). The advantage of ℓ_2 norm regularization is that the cost function minimum can be easily estimated by solving linear system of equations. However, ℓ_2 model regularization results in smooth (low resolution) Radon model since it assumes that the model coefficients follow Gaussian probability density function (Sacchi and Ulrych, 1995a). Since the Radon transform basis functions match reflection hyperbolas, an ideal Radon model should be sparse. This is the main idea of the high resolution (also called sparse) Radon transforms (Thorson and Claerbout, 1985; Sacchi and Ulrych, 1995a,b; Trad et al., 2003). Sparse Radon transforms use a regularization term that enforce sparsity (ℓ_1 norm) and a quadratic misfit term in the cost function to estimate Radon model. However, the quadratic ℓ_2 misfit term in the cost function is sensitive to erratic noise in the data. For data contaminated with erratic noise, Claerbout and Muir (1973) suggested replacing the conventional ℓ_2 misfit with ℓ_1 misfit function which is not sensitive to erratic noise in the data (Guitton and Symes, 2003; Ji, 2006, 2012; Li et al., 2012). In simultaneous source acquisition, it is common that high amplitude reflections of one source interfere with the low amplitude reflections of another source. Since the amplitude changes significantly in seismic data, the interferences cause large misfits (data outliers). Therefore, the Radon model estimated using quadratic ℓ_2 misfit are not accurate because of the ℓ_2 misfit sensitivity to outliers. Ibrahim and Sacchi (2013, 2014) showed that the robust misfit is more effective in removing interference noise while preserving the signal. In this work, we use Iteratively Re-weighted Least Squares (IRLS) algorithm to estimate sparse and robust ASHRT model. For more details on IRLS and robust Radon transform please refer to Ibrahim and Sacchi (2014).

Stolt operator

Stolt (1978) introduced a migration operator that map the temporal frequency ω to the vertical wavenumber k_z in Fourier domain for constant velocity using the dispersion relation

$$\omega = V\sqrt{k_x^2 + k_x^2}, \tag{3}$$

where V is the migration velocity and k_x is the horizontal wavenumber. This is followed by scaling the amplitude by

$$S = V\,\frac{k_z}{\sqrt{k_x^2 + k_z^2}}. \tag{4}$$

Therefore, the adjoint Stolt (migration) operator can be written as a sequence of three operators

$$\mathbf{L}^T = \mathbf{FFT}^{-1}_{k_z,k_x}\,\mathbf{M}^T_{\omega,k_x}\,\mathbf{FFT}_{t,x}, \tag{5}$$

and similarly the forward (de-migration) operator can be written as

$$\mathbf{L} = \mathbf{FFT}^{-1}_{\omega,k_x}\,\mathbf{M}_{k_z,k_x}\,\mathbf{FFT}_{z,x}, \tag{6}$$

where, $\mathbf{M}$ is the $f-k$ mapping operator and $\mathbf{FFT}$ is the Fast Fourier Transform operator. Although the Stolt operator is derived with constant velocity assumption, it can be used to construct ASHRT model with multiple velocities. We can compute the ASHRT model using several Stolt models with each representing one plane inside the ASHRT model cube as shown in Figure 2. The classical ASHRT operator has a computa-

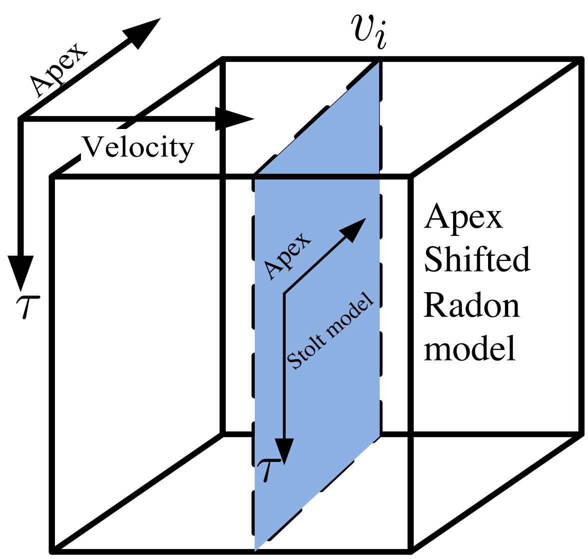

Figure 2: A Stolt model is a plane at constant velocity inside the ASHRT model cube.

tional cost of $O(n_a \times n_\tau \times n_v \times n_x)$, where n_a, n_τ, n_v and n_x are the number of apex locations, apex times, velocities and offsets, respectively. Assuming that we scan for all possible apex locations and times, then $n_a = n_x$ and $n_\tau = n_t$. Therefore, the ASHRT operator cost is $O(n_x^2 \times n_t \times n_v)$. On the other hand, the Stolt based ASHRT (without FFT zero padding) operator has a cost that is of the 2D FFT of the data with size $n_t \times n_x$ followed by $f-k$ mapping and inverse 2D FFT of the model with size $n_t \times n_v \times n_x$. Therefore, the total computational cost of an ASHRT implemented via the Stlot operator is $O([n_t \log_2(n_t) + n_x \log_2(n_x)][n_v + 1] + n_v \times n_{kx} \times n_\omega)$, where n_{kx} and n_ω are numbers of horizontal wavenumbers and temporal frequencies, respectively. The cost of the $f-k$ mapping is proportional to $n_v \times n_{kx} \times n_\omega$ and we stress that the latter is an upper limit, since in practice we only scan for a limited band of positive frequencies and use the Fourier domain symmetry to compute the negative frequencies. Figure 3a shows the computational times of the ASHRT and the Stolt operator with and without zero padding. Zero padding is sometimes required to reduce artifacts associated with $f-k$ interpolation. Figure 3b shows the improvement in the computational time by using the

Stolt operator with and without zero padding compared to the conventional ASHRT. It is clear that computing Radon model via the Stolt operator can lead to a significant saving in computational cost. This is very important for processing large data set with a large number of common receiver gathers.

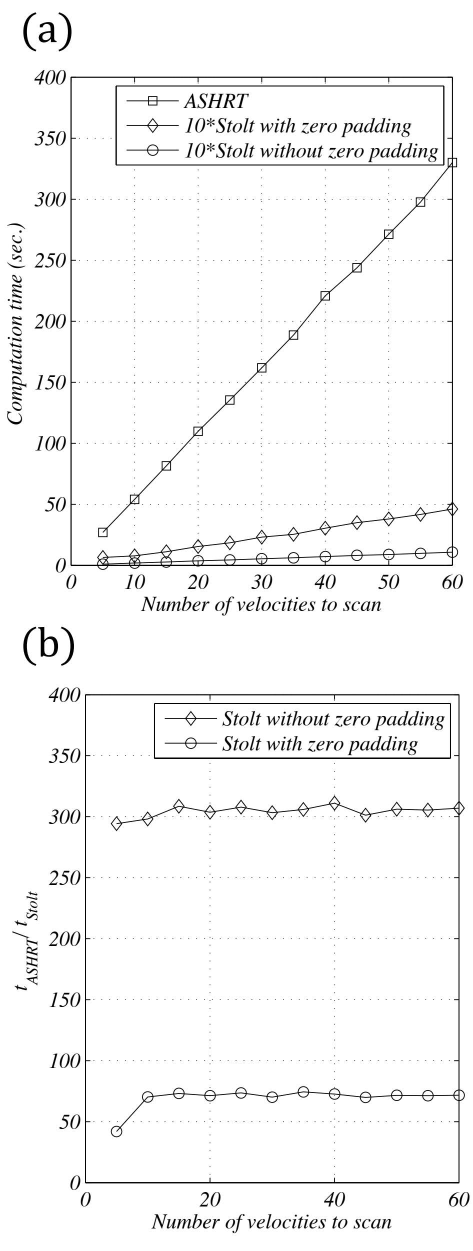

Figure 3: Comparing operators (a) Computation times. (b) The Stolt operator computation times compared to the ASHRT.

EXAMPLES

We tested the robust the Stolt-based ASHRT with a synthetic and a marine data set from the Gulf of Mexico. Both data sets are blended numerically with a 50% time reduction compared to the conventional acquisition. The blending scheme represents one source firing with random delays while the receivers are at fixed locations. For each of the two data sets, the blended data is initially separated using the pseudo-deblending operator. Then each common receiver gather is de-noised using robust inversion. Figure 4 and figure 5 show the results of the synthetic and real data examples, respectively. The quality of the recovered data is measured using the following expression

$$Q = 10Log\frac{\|\mathbf{d}_{original}\|_2^2}{\|\mathbf{d}_{original} - \mathbf{d}_{recovered}\|_2^2}. \quad (7)$$

The Q value for the recovered synthetic data cube is 22.01 dB and for the real data cube is 11.11 dB.

CONCLUSION

We have implemented a Radon transform designed via the Stolt operator to eliminate erratic incoherent noise that arises in common receiver gathers of simultaneous source data. We showed that the Radon transform based on the Stolt operator can remove source interferences in common receiver gathers at a computational cost that is substantially below the computational cost of the classical Radon transform. The Stolt operator is a more computationally efficient approach to the computation of the apex shifted Radon transform. Since the Stolt operator is implemented in $f-k$ domain, it can be used in combination with the non-uniform Fourier transform to interpolate missing traces. Future work entails generalizing the problem to the 3D shot distribution.

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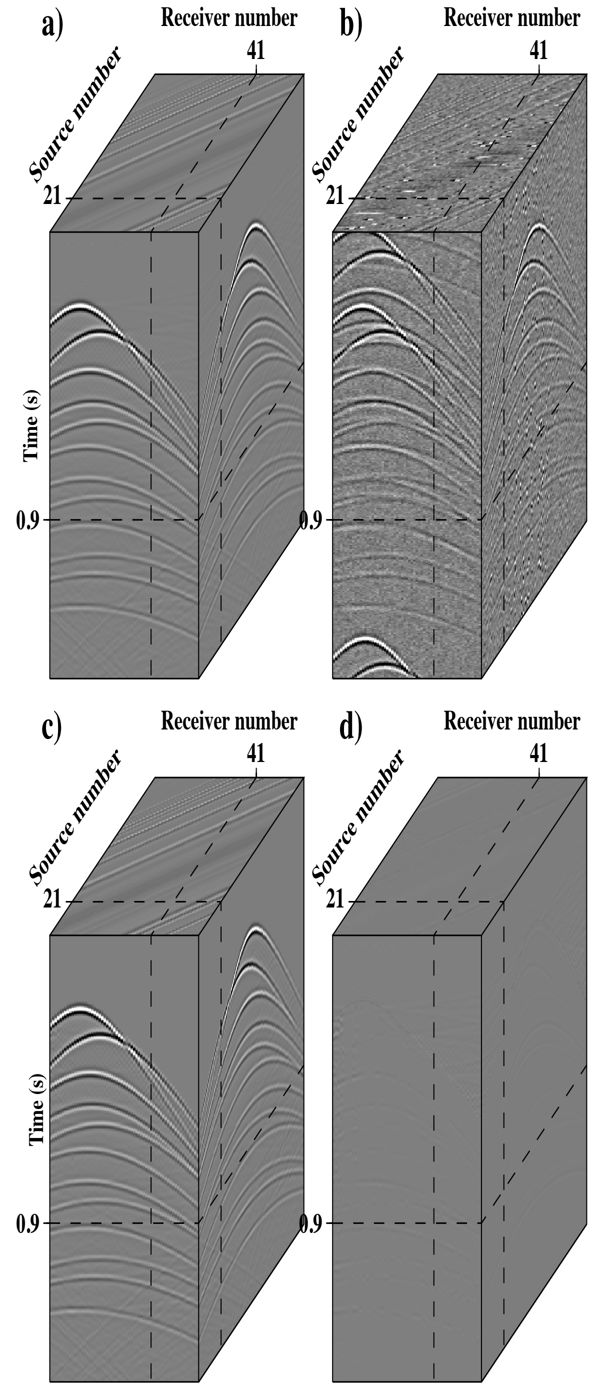

Figure 4: Synthetic data cube. (a) Original data . (b) Pseudo-deblended data. (c) Data recovered by forward modelling the Radon coefficients estimated via the robust Stolt $p = 1$ and $q = 1$. (d) Difference between the recovered and the original data cubes.

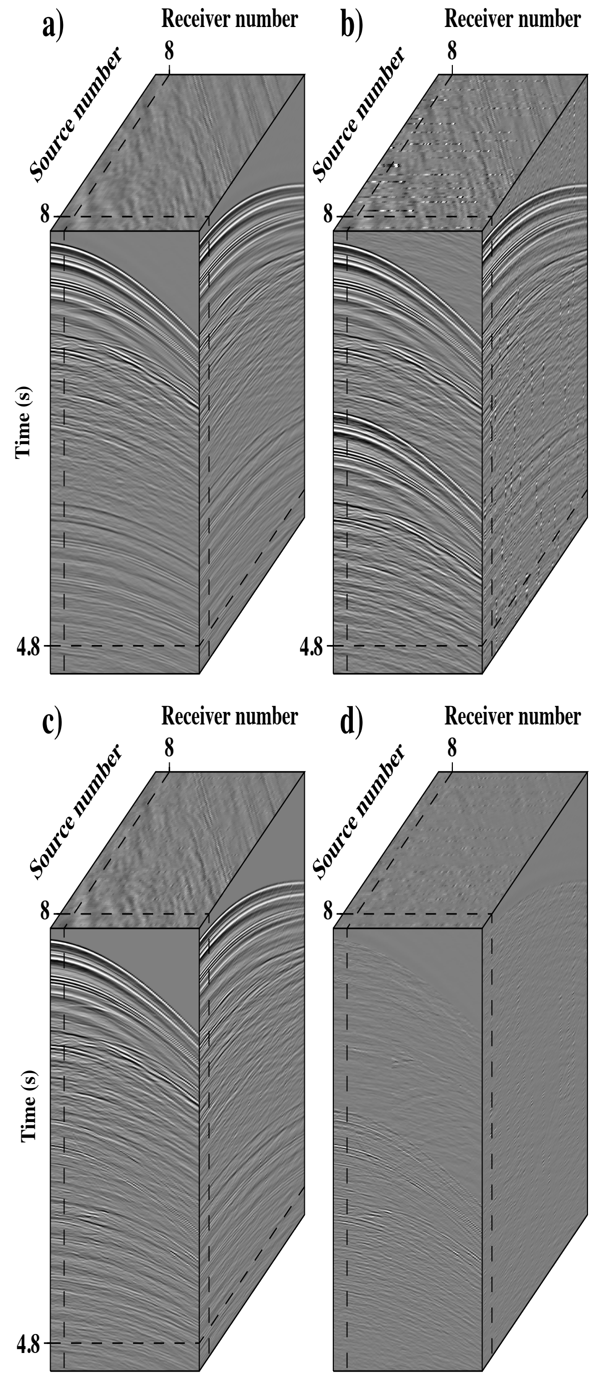

Figure 5: Real data cube. (a) Original. (b) Pseudo-deblended. (c) Data recovered by forward modelling the Radon coefficients estimated via the robust Stolt $p = 1$ and $q = 1$. (d) Difference between the recovered and the original data cubes.

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