Separation of simultaneous source data via iterative rank reduction

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SUMMARY

In this paper, we report an inversion algorithm based on singular spectrum analysis (SSA) that is capable of suppressing the interferences generated by simultaneous source acquisition. We derive an iterative scheme that adopts the projected gradient method to solve the source separation problem. The projection operator is the SSA rank reduction filter that suppresses incoherent noise in the frequency-space domain. Convergence of this algorithm can be achieved with appropriate choice of step size and an initial starting point. We use synthetic examples simulated with a data set from the Gulf of Mexico to illustrate this method.

INTRODUCTION

Simultaneous source acquisition has been attracting much attention because of the great economical potential it offers. In the configuration of simultaneous source acquisition, instead of firing one shot at a time and imposing large time intervals between shots, several shots fire at close times (Beasley et al., 1998). The major problem associated with this acquisition design is the strong interferences caused by having seismic events originating from more than one shot recorded by arrays of receivers. Randomization of time delays would make interferences appear incoherent in domains like common receiver gathers (Stefani et al., 2007). This fact further leads to a variety of separation algorithms. Direct processing steps like pre-stack time migration and stacking are considered sufficient to suppress the overlaps in 3D acquisition (Krey, 1987; Stefani et al., 2007). However, problems may rise when performing amplitude sensitive analysis, such as AVO inversion and time-lapse seismic monitoring. Coherent pass filters in $\tau - p$, $f - k$ and $f - x$ domains were developed to filter out incoherent interferences (Akerberg et al., 2008; Moore et al., 2008; Huo et al., 2009; Maraschini et al., 2012). Better separation was achieved with inversion methods based on projecting the data to other domains where the coherent constraints can be effectively implemented (Abma et al., 2010; Mahdad et al., 2011; Lin and Herrmann, 2009; Mansour et al., 2012).

In this paper, we present a method named iterative rank reduction (IRR) for separating simultaneously fired shots before stacking. This method is basically an inversion method and relies on the incoherency of overlaps in common receiver domain.

THEORY

Preliminaries

We use the matrix representation of seismic data presented by Berkhout (2008), where data acquired from traditional acquisition methods can be arranged into the so-called data matrix D. Each row of D represents a shot record and each column corresponds to a receiver gather. Then data acquired from simultaneous source acquisition D^{obs} can be expressed by

$$D^{obs} = \Gamma D\,, \tag{1}$$

where Γ is the blending operator. This system of equations is under-constrained as the signal received in each detector contains information from multiple sources. The minimum norm solution would exactly lead to the adjoint operator Γ^* also called the pseudo-deblending operator. The latter implies the process of shifting time delays back and decomposing the blended shot into conventional shot gathers. Unfortunately, we cannot recover the unblended shot without applying interference removal techniques.

Iterative Rank Reduction

Let's consider Equation (1) as a linear projection. In addition, we consider that the ideal data D can be represented via a low rank matrix. As we will see this is not a good assumption, however, it will permit to develop the basic Iterative Rank Reduction algorithm which will be used to separate blended shots. Our problem is to estimate D via the solution of the following optimization problem

$$minimize \quad J = \|D^{obs} - \Gamma D\|_2^2 \quad s.t. \quad Rank(D) = k. \tag{2}$$

Equation (2) can be solved via singular value decomposition (SVD). However, we can also adapt the classic Landweber iteration and solve the problem in an iterative framework as follows

$$\begin{aligned} x_{i+1} &= D_i - \lambda \nabla J \\ D_{i+1} &= P(x_{i+1}), \end{aligned} \tag{3}$$

where P is an operator that projects data into a low rank matrix. This can be effectively done utilizing SVD, Lanczos decomposition or low rank matrix factorization techniques. The gradient is given by

$$\nabla J = \Gamma^*(\Gamma D_i - D^{obs}) \tag{4}$$

In each iteration, we minimize the misfit function by updating the current model solution along the gradient descent direction. The solutions are then projected to a matrix of low (known) rank. This algorithm, known as fixed point iterative algorithm or singular value projection, has been discussed in details with examples in matrix completion (Ma et al., 2011; Jain et al., 2009). The convergence of this algorithm holds

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when the stepsize, $\lambda < 2/\sigma_{max}$ with σ_{max} the maximum eigenvalue of the operator $(\Gamma^*\Gamma)$. In expression (3) the operator $P[x]$ symbolized for instance rank reduction via the SVD.

Iterative Rank Reduction with SSA denoising

In our approach, we replace the constraint $Rank(D) = k$ by $Rank(H) = k$ where H is the Hankel matrix computed from spatial data at a given monochromatic frequency. In other words, the low rank data constraint was replaced by a Cadzow filter also called the Singular Spectrum Analysis (SSA) denosing method (Sacchi, 2009; Trickett and Burroughs, 2009; Oropeza and Sacchi, 2011). SSA denoising can be expressed by the following algorithm

Algorithm 1 Singular Spectrum Analysis (SSA) denoising: F_{ssa}

Inputs:
  Data to denoise (preserve linear events): D
  Size of subspace: k;
$d^{\omega-x} \leftarrow \mathscr{F}_t(D)$
for each ω **do**
  $H \leftarrow Hankel(d^{\omega-x})$
  Compute largest k singular values and associated singular vectors of H: U_k, Σ_k, V_k
  $H_k \leftarrow U_k \Sigma_k V_k^H$
  $d_k^{\omega-x} \leftarrow$ averaging along antidiagonals of H_k
end for
$\hat{D} \leftarrow \mathscr{F}_t^{-1}(d_k^{\omega-x})$
return $\hat{D}$: the SSA denoised data

SSA entails forming Hankel matrices from $f-x$ domain data, performing rank reduction and then recovering the data via anti-diagonal averaging of the reduced-rank Hankel form. It can be shown that SSA can preserve linear events and filter incoherent events (Sacchi, 2009; Trickett and Burroughs, 2009). In addition, it is easy to show that for a data set composed of the superposition of k dips, the Hankel matrix of the data is a rank k matrix. If the data are contaminated with noise, the rank of their associated Hankel matrix will increase and therefore, rank reduction is an effective way of noise attenuation. One way to exclude interferences caused by secondary sources is by filtering in small windows of data in the common-receiver domain with the SSA filter. By doing this, we are assuming that the events are predictable along the spatial direction. Then the rank constrained optimization problem can be reformulated as follows

$$minimize \quad J = \|D^{obs} - \Gamma D\|_2^2 \quad s.t. \quad Rank[H(D)] = k, \qquad (5)$$

where H represents projecting data to a Hankel structured matrix. We adapt the solver used for Equation (2) by replacing the SVD with SSA. Now P is a new projection operator that contains the processes of sorting data into common receiver gathers, windowing, SSA filtering and resorting data to common source domain. The adapted iterative rank reduction method is shown in Algorithm (2). The projection operator P is now given by

$$P[x] = S^* \sum_i F_{ssa} W_i S[x] \qquad (6)$$

where S denotes sorting in common receiver gathers, W_i is the localized $t-x$ small i-th window in common receiver domain, F_{ssa} is the aforedescribed SSA denoising filter and S^* means sorting back to common source gathers after window patching. The window functions W_i are designed with overlaps that honor a partition of unity $\sum_i W_i = 1$.

Algorithm 2 Iterative Rank Reduction with SSA denoising

Inputs:
  Blending and its adjoint operator Γ and Γ^*
  Observed blended data D^{obs}
  Size of subspace k
  Stopping criterion ε;
  Initial stepsize λ_0
Initialize:
  $D^0 = \Gamma^* D^{obs}$; $i = 0$;
repeat
  $\lambda = \lambda_0/\sqrt{i}$
  $x_{i+1} \leftarrow D_i - \lambda \Gamma^*(\Gamma D_i - D^{obs})$
  $D_{i+1} \leftarrow P(x_{i+1})$
  $i \leftarrow i+1$
until $\|D_{obs} - \Gamma D\|_2^2 < \varepsilon$

It is very difficult to prove the convergence of this algorithm because the projection operator $P[x]$ is non-convex. The solution to the iterative algorithm can be trapped into local minima if the initial point and step size are not properly selected (Fazel, 2002). In this paper, a reasonable candidate for initial solution is the pseudo-deblended data. This is because the pseudo-deblended data contain exactly the signals from an unblended shot and it is actually very close to the true solution. Now the problem becomes suppressing noise in the pseudo-deblended dataset by performing iterative rank reduction. Convergence can then be achieved with the nonsummable diminishing step lengths, which is shown in Figure (1). In this example the step size is decreased according to $1/\sqrt{(iteration)}$. We show the misfit values between solution and observation in blended domain, as well as the differences with respect to the true solution in unblended domain. The algorithm is comparatively effective as both curves reach convergence after about 20 iterations. We can also let the rank increase with iterations. At early iterations, we can apply harsh filtering to eliminate strong crosstalk and gradually increase the rank to allow modeling details that depart from the linear event model. This analogous to setting the threshold schedule in projection-onto-convex sets regularization and deblending methods (Abma et al., 2010).

EXAMPLES

We test the proposed algorithm with a 2D synthetic example simulated from field dataset acquired in the Gulf of Mexico. The spatial and temporal sampling intervals are set to be 4ms and 87.5 feet. Each time we assume 5 shots that fire almost simultaneously and 16 supershots are generated. The firing time

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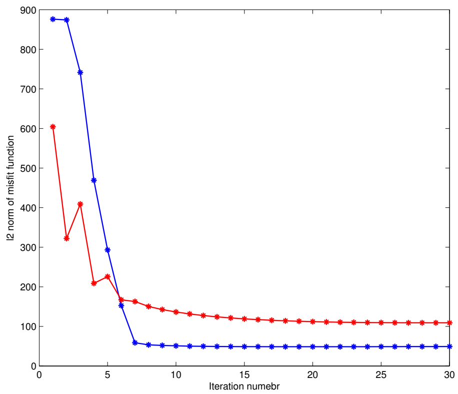

Figure 1: Convergence of iterative rank reduction source separation algorithm. Blue line indicates the l_2 norm of the difference between the blended observations and the synthesized blended observation versus iteration. We also portray in blue the difference between the unblended data and the true data versus iteration.

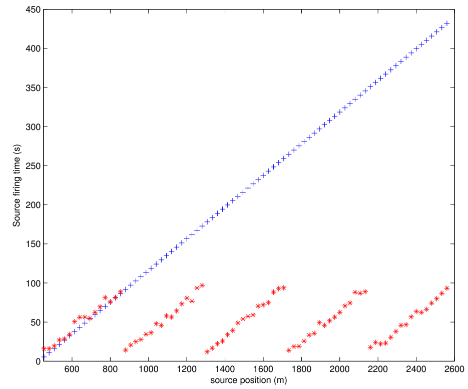

Figure 2: Temporal and spatial source distribution of simultaneous source separation. Each time 5 shots are recorded by a same set of receivers

are random. Figure (2) shows the spatial and temporal distribution of sources in this acquisition design. Then we apply the proposed algorithm to unblend the data for the whole volume. Figure (3) shows the results of the separation after 40 iterations for shot number 20. The interferences from simultaneously fired shots are effectively suppressed. We improve the signal to noise ratio of the pseudo-deblended dataset from 0 dB to 7.78 dB. As a result, the unblended solution becomes comparable with the true shot record. In this example the rank parameter was variable with iteration number, the SSA filter was run in overlapping windows in common receiver gathers of size 100 time samples and 20 traces. The windows were overlapped 20% in both time and space with Gaussian tappers designed to satisfy a partition of unity.

We have tested the algorithm under diverse parameters and noticed that small windows are needed when operating with small subspaces. Increasing the rank of the SSA filtering with progressing iterations helps to avoid having harsh filters all way in the optimization process. A strategy that have work for us entailed starting with $k = 3$ and increased the rank by one unit every 5 iterations. In short, the final rank after 30 iterations was 9.

CONCLUSION

This paper illustrates an iterative rank reduction algorithm based on singular spectrum analysis for separating simultaneous sources. The proposed algorithm can be classified among the family of deblending methods via inversion schemes. By implementing rank reduction with a projection operator via the SSA filter, solutions are constrained to be low rank in Hankel matrices extracted from small spatio-temporal windows in common receiver domain. The latter is important because the SSA method is a valid denosing technique for a superposition of plane waves. In a small window the data can be approximated via a limited number of dips plus incoherent interferences caused by the blending process. Convergence of this algorithm can be achieved if the pseudo-deblended data is adopted as the initial solution. Through tests with synthetic examples made by blending a traditional marine acquisition, we show that the interferences of the wavefields can be effectively suppressed. This algorithm also sees applications in multidimensional cases by adopting high order SVDs (Kreimer and Sacchi, 2012) or by adopting block Hankel matrices (Gao et al., 2011).

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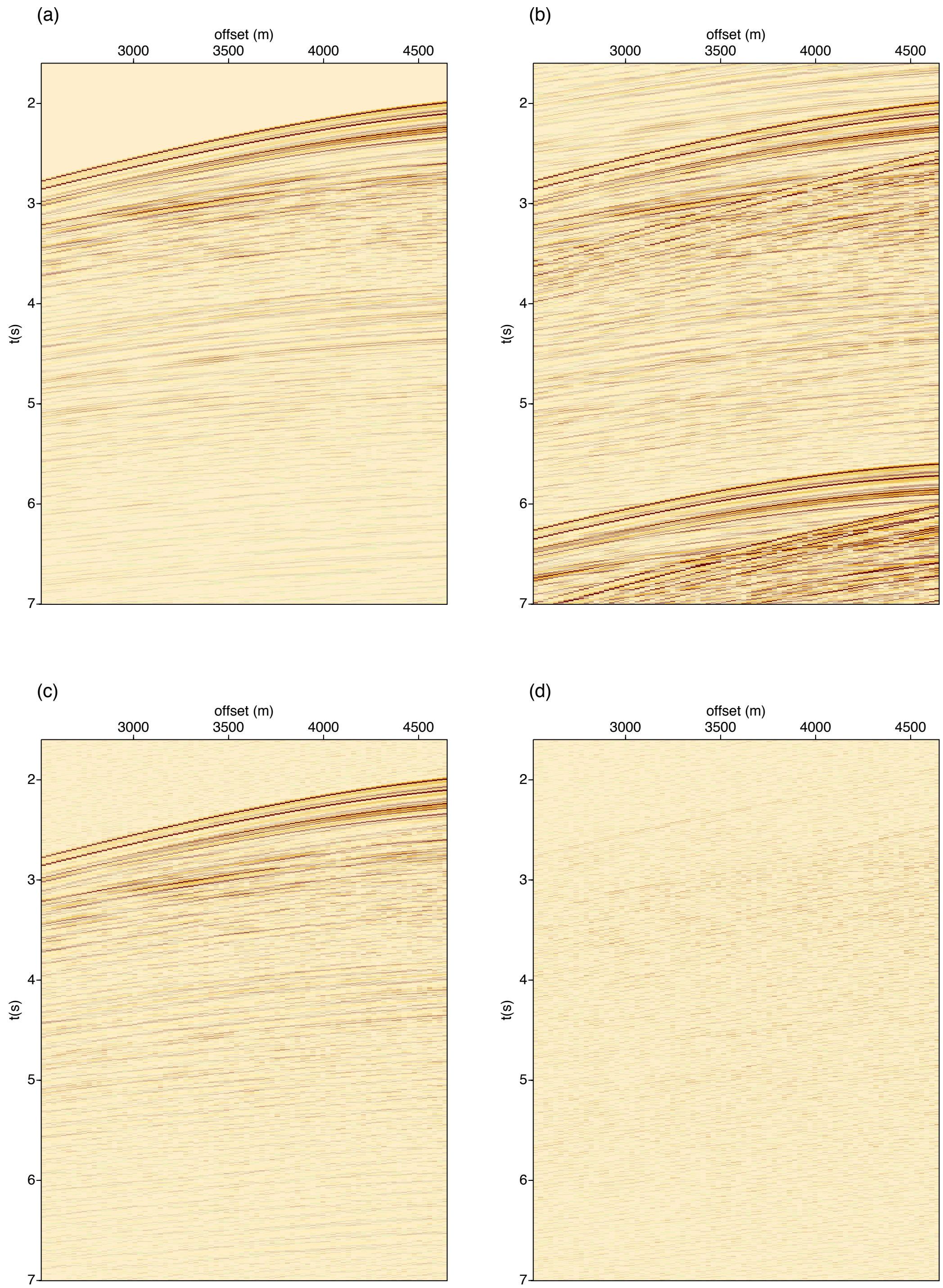

Figure 3: Results of separation via Iterative Rank Reduction method: (a) The real unblended shot record (20th). (b) Pseudo-deblended shot record. (c) Shot record separated with singular value projection. (d) Differences between (a) and (c)

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EDITED REFERENCES

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