Optimization for sparse acquisition

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SUMMARY

Acquisition design plays a significant role in seismic exploration and data processing. An optimized seismic acquisition design will require fewer resources and therefore, it can reduce the total cost of seismic exploration. Finding the optimal locations of sources and receivers in a seismic survey is a long-standing problem. Random sampling can recover high-bandwidth of seismic data using fewer number of sensors, however, it might not be viable to implement in real acquisition scenarios. In this paper, we have proposed a technique to determine the optimal distribution of sources and receivers for an orthogonal 3D seismic survey while satisfying geophysical, operational, and reconstructional constraints for a real acquisition design. The proposed sampling method adopts the concept from the field of compressive sensing (CS) that connect acquisition design and data reconstruction. During 5D reconstruction, seismic data are generally transformed from source-receiver domain into common midpoint (CMP)-offset domain after binning. Therefore, to implement CS successfully, we require to minimize the mutual coherency of every data patch in CMP-offset domain while satisfying other geophysical and operational constraints in the source-receiver domain. This is a complex non-linear problem and hence, we have implemented simulated annealing (SA) with Augmented Lagrangian method to solve this problem (Powell (1969); Belegundu and Arora (1984)).

INTRODUCTION

Different geophysical and operational parameters such as bin size, maximum offset, largest minimum offset (LMOS), field geometry, and available resources (i.e., sources and receivers) have significant impact in seismic acquisition design (Cordsen et al., 2000). In conventional seismic data acquisition design, the field geometry is generally assumed dense and orthogonal not only to avoid spatial aliasing artifacts but also to obtain high-fidelity and high-resolution seismic data (Kerekes, 1998). Nevertheless, this classical acquisition technique drastically increase the total cost of the survey and impose adverse impact on environment both in land and marine scenarios. A number of methodologies have already been developed to solve this optimization problem. Liner et al. (1999) introduced a method which optimize the geophysical parameters of 3D acquisition design. Later on, this method has been modified and improved by Morrice DJ (2001) and Vermeer (2003) where they included economical and operational constraints in that cost function. However, in this paper, we have proposed to design a sparse acquisition with reduced number of sources and receivers which entails to satisfy not only geophysical and operational constraints but also constraints related to seismic data reconstruction. Several methods have been proposed to optimize survey design based on data reconstruction and imaging. These methods include optimizing signal to noise ratio (SNR) of the reconstructed data (Mosher et al., 2012), forward modelling of seismic wave-fields and imaging (Zhu et al., 2012) etc.

Compressive Sensing (CS) is a mathematical tool which permits a compressible signal to be recovered exactly from a set of measurements that are far fewer than the Nyquist sampling rate (Donoho, 2006). A fundamental requirement to implement this technique in geophysics is to having a sparse representation of seismic data in some domain (Candes and Walkin, 2008). Fortunately, plenty of algorithms (e.g., Zwartjes and Sacchi (2007); Hennenfent and Herrmann (2008) etc.) have been developed that utilize the sparsity characteristics of seismic data that makes easier to implement CS in seismic survey design. Regular decimation of measurements creates aliasing in spatial domain of the original signal that makes sparse-inversion failed. In contrast, random-like sampling in a grid or patch can distribute noise-like incoherent energy in frequency-wavenumber domain which is favourable for available data reconstruction methods (Trad, 2009). Based on recent CS results (Elad, 2007), a sampling scheme can be optimally designed whereby the accuracy of reconstruction can be improved or the number of required samples can be reduced. Several authors (e.g., Elad (2007); Tang et al. (2008)) defined mutual coherency as a criterion for the accuracy of reconstruction. Hennenfent and Herrmann (2008) proposed jitter under-sampling technique in receiver domain for 2D acquisition design that can control gap size up to a certain extent, however, unable to satisfy logistic constraints. In this paper, we optimized 3D survey design which satisfies geophysical and operation constraints in source-receiver domain and simultaneously, minimize mutual coherency of 5D patches in CMP-offset domain for the betterment of data reconstruction.

THEORY

According to the theory of compressive sensing, seismic data $\mathbf{x}$ can be reconstructed from undersampled observed data $\mathbf{y}$ via exploiting any sparsity-promoting non-linear convex optimization algorithm (Elad (2007); Donoho (2006)). Mathematically, this under-determined system of equations can be expressed as follows

$$\begin{aligned} \mathbf{y} &= \mathbf{R}\mathbf{x} + \varepsilon \qquad (1) \\ &= \mathbf{R}\mathbf{S}\alpha + \varepsilon, \qquad (2) \end{aligned}$$

where, ε is the measurement noise, $\mathbf{R}$, the sampling operator that maps from the original data to observed data, $\mathbf{S}$ is a transformation basis or dictionary such as Fourier, Wavelets, Curvelet, or Wave-atom, and α is the vector of coefficients that represents $\mathbf{x}$ as a linear combination of $\mathbf{S}$. Undersampled data can be reconstructed successfully using CS theory if one can satisfy two constraints: sparsity and incoherence. The data $\mathbf{x}$ is said to be sparse in dictionary $\mathbf{S}$ if a very few coefficients

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of α are non-zero. Fortunately, seismic data can be represented as sparse in Fourier and other domain (Sacchi (2009); Hennenfent and Herrmann (2008)). Second condition in CS theory is that the coherence between sampling matrix ($\mathbf{R}$) and transformation dictionary ($\mathbf{S}$) should be as small as possible. Mutual coherency is a way to verify this constraint. The mutual coherency of $\mathbf{A}$ (i.e., $\mathbf{RS}$) can be represented by using the Gramian matrix which is expressed using equation (3).

$$\mathbf{G} = \mathbf{A}^H\mathbf{A}, \quad (3)$$
$$= \mathbf{S^H R^T R S}. \quad (4)$$

In this paper, we consider $\mathbf{S}$ as a Fourier operator applied in 4D spatial directions (i.e., inline CMP, crossline CMP, and offset in inline and crossline directions) in CMP-offset (Ω_y) domain. Therefore, we have to deduce an optimized sampling in the Ω_y grid so that the mutual coherency becomes very small. To minimize the mutual coherency of $\mathbf{A}$, the Gramian matrix needs to be close to an identity matrix.

Seismic data are recorded in source-receiver domain during 3D seismic survey. However, prior to 5D data reconstruction, seismic data is transformed from source-receiver domain (Ω_x) into CMP-offset domain (Ω_y). Subsequently, data are binned in CMP-offset domain using nearest neighbour interpolation which is a non-invertible process. The whole process can be represented as

$$\Omega_y = \mathscr{B}\mathscr{G}\Omega_x, \quad (5)$$

where $\mathscr{G}$ is an operator which transform source-receiver domain to CMP-offset domain, and $\mathscr{B}$ is the binning operator which applies to CMP-offset domain to obtain binned data. Consider now that one would like to reconstruct 5D data in Ω_y grid and hence divide the whole grid into small 5D patches and reconstruct each patch individually. Therefore, it will require to optimize the mutual coherency of patches while satisfying other acquisition design parameters in Ω_x domain. The series of mutual coherency of 5D patches can be defined as $\{\mu_y\}$ and μ_{y_i} is the mutual coherency of ith patch. To optimize the mutual coherency of every patch, we minimize both the average mutual coherency and the quantity of critical patches (ρ_μ) where mutual coherency is very high. ρ_μ can be computed applying simple statistical analysis of $\{\mu_y\}$. Mathematically, it can be expressed as follows:

$$\begin{aligned} \mu_n^{std} &= \frac{\mu_{y_i} - \mu_{\mathbf{y}_{ave}}}{\sigma_{\mu_y}} \\ \hat{\mu}_n^{std} &= \mu_n^{std} - Var(\mu_n^{std}) \\ \rho_\mu &= \frac{\Gamma(\hat{\mu}_n^{std})}{N} \end{aligned} \quad (6)$$

where σ_{μ_y}, $\mu_{\mathbf{y}_{ave}}$, and μ_n^{std} are the standard deviation, average mutual coherency, and normalized standard deviation of the series $\{\mu_y\}$ respectively, Γ operator counts the number of patches having higher mutual coherency than the variance of normalized standard deviation (μ_n^{std}), and N represents the total number of patches. The desired value of fraction of critical patches ($\bar{\rho}_\mu$) is zero. In this paper, we also satisfy other reconstructional parameters such as grid efficiency (η_y) and grid density (g_y). Grid efficiency can be defined as the ratio of the number of bins that are populated in (Ω_y) domain and the number of traces generated in (Ω_x) domain. Our target value for grid efficiency ($\bar{\eta}_y$) is 1.0. On the other hand, grid density (g_y) is the ratio of the number of populated bins to the total number of bins in (Ω_y) domain. Based on CS theory, sparsity-promoting algorithm can reconstruct a volume that is populated by 10-15% of traces having set that mutual coherency is smaller and sparsity is higher. Likewise mutual coherency, we optimize grid density for every patch in (Ω_y) domain via exploiting average grid density ($g_{y_{ave}}$) and minimizing the fraction of critical patches where grid density (ρ_g) is very low. The target values of $\bar{g}_{y_{ave}}$ and $\bar{\rho}_g$ are 0.15 and 0 respectively.

Regarding geophysical and operational constraints, we optimize bin size in crossline and inline directions, maximum offset, largest minimum offset (LMOS), and available resources during operation. Bin size is a crucial parameter which dictates source (SI) and receiver (RI) interval and itself depends on three parameters: minimum target size, maximum unaliased frequency, and lateral resolution based on the first fresnel zone after migration. Source (SLI) and receiver (RLI) line interval is dictated by LMOS which is computed based on the shallowest target horizon of the subsurface (i.e., 2 $\times$ depth of shallowest horizon$\times$ tan(35^0)). The desired value of maximum offset depends on the depth of the deepest horizon that needs to be illuminated. It is also required for AVO analysis, and DMO for velocity determination (Cordsen et al., 2000).

In this paper, we consider bin size is fixed as target value and we have enough available resources for the whole survey design. Therefore, putting it all together, we now have a problem where we need to minimize a cost function subject to constraints. Here one wants to know the source and receiver distribution in Ω_x not only to ensure the proper distribution in Ω_y but also to meet the constraints in the survey domain. Mathematically, the cost function can be written as follows:

$$\begin{aligned} &\underset{\Omega_x}{\text{argmin}}\ \mu_{\mathbf{y}_{ave}}(\Omega_x) \\ &\text{subject to} \\ &\eta_y - \bar{\eta}_y \approx 0 \\ &\rho_\mu - \bar{\rho}_\mu \approx 0 \\ &\rho_g - \bar{\rho}_g \approx 0 \\ &X_{max} - \bar{X}_{max} \geq 0 \\ &X_{min} - \bar{X}_{min} \leq 0 \\ &g_{y_{ave}} - \bar{g}_{y_{ave}} \geq 0 \end{aligned} \quad (7)$$

where X_{max}, and X_{min} represent maximum offset, and largest minimum offset respectively, and $\bar{X}_{max}$, and $\bar{X}_{min}$ are the target values of maximum offset, and largest minimum offset respectively. To proceed, we have started with an orthogonal source and receiver geometry with regular interval. Initially, (SI, RI) and (SLI, RLI) are computed based on bin size and shallowest horizon of subsurface respectively. Subsequently, we decimate sources and receivers, and start perturbation ($\Omega_x + \Delta\Omega_x$) in such a way so that it can control the maximum gap and represents the feasible set of shots and receivers distribution. To minimize the cost function in equation (7), we applied hybrid semi-exhaustive search method using simulated annealing (SA) with Augmented Lagrangian method Wah et al. (2007).

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Each constraint in the cost function is identified as hard or soft. Hard constraint needs to be satisfied precisely and a soft constraint only requires to be close to the target value. X_{min}, X_{max} are soft constraints, however, $\Omega_x + \Delta\Omega_x$ is the hard constraints. Regarding reconstructional parameters $g_{y_{ave}}$, η_y, ρ_μ, and ρ_g are soft constraints which ensure quality data reconstruction for each 5D patch. The above constrained optimization problem is transformed into unconstrained optimization function using method of multipliers technique. This method can accommodate both equality and inequality constraints (Powell (1969); Hestenes (1969)). These constraints are normalized to equalize the deviation from target values, to avoid numerical instability and to improve the optimization performance.

$$
\begin{aligned}
\underset{\Omega_x}{\operatorname{argmin}}\, J(\Omega_x, \sigma, \theta) = \mu_{\mathbf{y}_{ave}}(\Omega_x) + \frac{1}{2}\Bigg(& \sigma_1 \left((\rho_\mu - \bar{\rho}_\mu) + \theta_1\right)^2 + \sigma_2 \left((\rho_g - \bar{\rho}_g) + \theta_2\right)^2 \\
& + \sigma_3 \left(\frac{\bar{\eta}_y - \eta_y}{\bar{\eta}_y} + \theta_3\right)^2 + \sigma_4 \left[\left(\frac{\bar{x}_{min} - x_{min}}{\bar{x}_{min}} + \theta_4\right)^-\right]^2 \\
& + \sigma_5 \left[\left(\frac{\bar{x}_{max} - x_{max}}{\bar{x}_{max}} + \theta_5\right)^+\right]^2 + \sigma_6 \left[\left(\frac{\bar{g}_{y_{ave}} - g_{y_{ave}}}{\bar{g}_{y_{ave}}} + \theta_6\right)^+\right]^2 \Bigg),
\end{aligned}
\tag{8}
$$

$$
\underset{\Omega_x}{\operatorname{argmin}}\, J(\Omega_x, \Sigma, \theta) = \mu_{\mathbf{y}_{ave}}(\Omega_x) + \frac{1}{2}(\mathbf{c} + \theta)^T \Sigma (\mathbf{c} + \theta)
\tag{9}
$$

where $\mathbf{c}$ is the vector consists of normalized constraints, σ_i and θ_i are parameters associated with ith constraint ($i = 1, 2, \cdots, 6$), and

$$
\begin{aligned}
(a)^+ &= \quad max(a, 0), \\
(a)^- &= \quad min(a, 0)
\end{aligned}
\tag{10}
$$

are used for inequality constraints. Σ is a diagonal matrix where σ_i's are the diagonal elements. External and internal iterations have been applied to solve the above cost function. Internal iterations are used to optimize source-receiver distribution (Ω_x) via exploiting SA algorithm and in external iterations, Σ and θ are changed to minimize the unconstrained cost function $J(\Omega_x, \Sigma, \theta)$. The parameter θ has been introduced by Powell (1969) to eliminate the requirement of Σ to reach infinity. At each external iteration, θ is increased for the violated constraints to force them into satisfaction. However, vector Σ is increased to speed up the rate of convergence of the algorithm.

RESULTS

A three layered simple subsurface model has been generated to run the simulation to optimize acquisition design. The survey size is 1.2 Km × 1.2 Km and the depth of the model is 1 Km. The maximum, average, and minimum velocities of the model are 3000 (m/s), 2400 (m/s), and 1750 (m/s) respectively. The maximum and dominant frequencies of source is chosen as 20 Hz and 40 Hz respectively. The depth of the shallowest horizon of the model to be illuminated is 250m. Based on all the parameters, the targeted values of bin size, maximum offset, and LMOS is fixed as 10 m both in inline and crossline directions, 1.4 Km, and 340 m respectively and determine initial source and receiver interval as 20 m and source and receiver line interval as 80 m and 40 m respectively. Therefore, the complete survey requires 15 and 30 source and receiver lines respectively, and each line consists of 60 stations in it. Figure 1 represents complete orthogonal geometry of source-receiver distribution. Before running simulation, we regularly decimated 50% of source and receiver lines and then start perturbing source and receiver lines both in inline and crossline directions respectively. We consider that every source is listened by every receiver of the acquisition geometry. Figure 2 shows the source-receiver distribution after optimization.

Figure 3 shows the convergence of the Augmented Lagrangian algorithm. One can see from this figure that fitness value is gradually decreasing with the number of external iterations of the algorithm. In each external iteration, the cost function is minimized using SA method. Figure 4 depicts the convergence of average mutual coherency of all patches. The series of mutual coherency obtained at the final iteration for 36 4D spatial patches has been shown in Figure 5. We can see from this figure that the mutual coherency for every patch of CMP-offset domain has been minimized. Grid efficiency has also been improved from 0.77 to 0.84 which is close to desired value (1.0) and average grid density of all patches is 0.14, close to 0.15. Maximum offset of the optimized acquisition geometry is 1.6 Km which is higher than desired value (1.4 Km). Finally, the largest minimum offset of the optimized source-receiver domain is 268.348 m which is smaller than 340 m.

CONCLUSIONS

In this paper, we have proposed a technique to determine the optimal distribution of sources and receivers for an orthogonal 3D seismic survey while satisfying geophysical, operational, and reconstructional constraints of real acquisition design. We have optimized mutual coherency, grid density, and grid efficiency to ensure better data reconstruction quality. We also considered maximum and largest minimum offset constraints to ensure the required illumination of the subsurface. Regularly decimated sources and receivers pattern has been perturbed in a predefined nominal grid via satisfying maximum gap constraints to ensure the viability to implement in real acquisition scenarios. The numerical results show that mutual coherency and fitness values have been minimized during the course of SA with Augmented Lagrangian algorithm. Further extension of this research will be considering fold distribution in the CMP-offset domain, binning error, swath shooting strategy when limited resources are available during acquisition, and comparing with other types of sampling distribution in the source-receiver domain.

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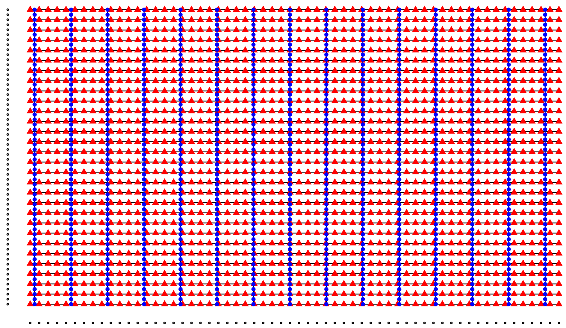

Figure 1: Fully sampled source-receiver distribution.

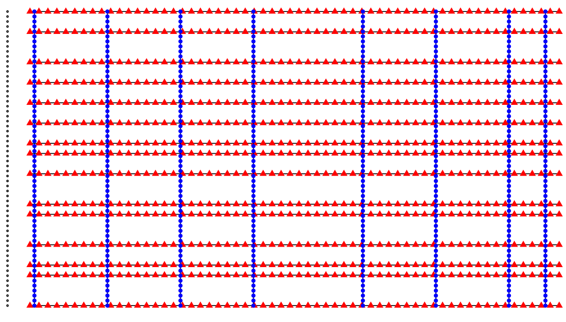

Figure 2: Optimized source-receiver distribution after decimation.

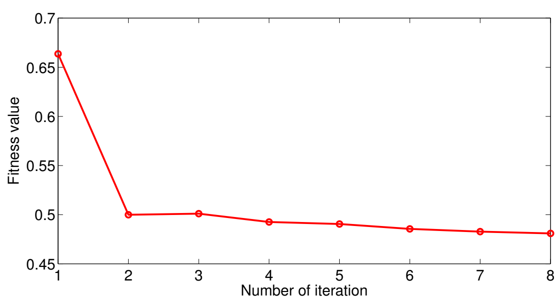

Figure 3: Convergence of fitness value.

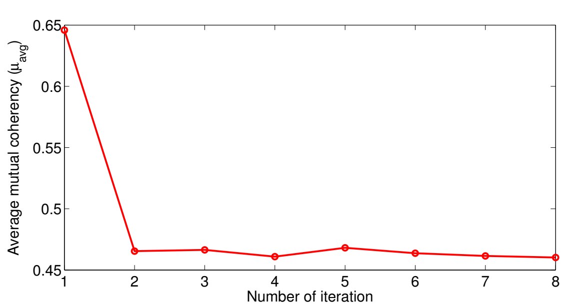

Figure 4: Convergence of average mutual coherency.

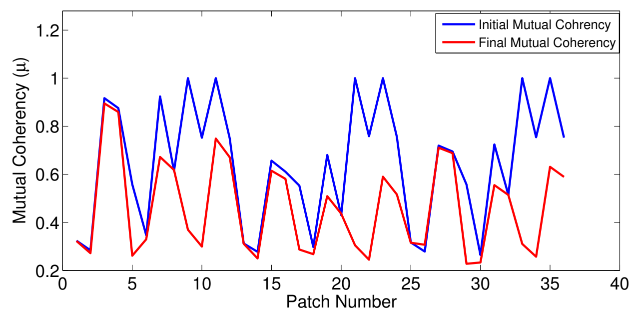

Figure 5: Distribution of initial and final mutual coherency for different patches.

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EDITED REFERENCES

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