A fast rank-reduction algorithm for 3D deblending via randomized QR decomposition

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SUMMARY

This paper illustrates an inversion approach based on matrix rank reduction that separates simultaneous source data. The algorithm operates on each common receiver gather of a multidimensional data set. We propose to minimize the misfit between the observed data and blended predicted data subject to a low-rank constraint that is applied to the data in the frequency-space domain. The low rank constraint can be implemented via the classical truncated Singular Valued Decomposition (tSVD) or via a new randomized QR decomposition (rQRd) method. Compared to the tSVD, rQRd significantly improves the computational efficiency of the method. In addition, the rQRd algorithm is less stringent on the selection of the rank of the data. This is important as we often have no precise knowledge of the optimal rank that is required to represent the data. We adopt a synthetic 3D VSP data set to test the performance of the proposed deblending algorithm. Through tests under different survey time ratios, we show that the proposed algorithm can effectively eliminate interferences caused by simultaneous shooting.

INTRODUCTION

Simultaneous source acquisition, or blended acquisition, has been attracting a great deal of attention because of the economic potential it brings to seismic data acquisition (Beasley et al., 1998; Berkhout, 2008). The technique aims at improving the acquisition efficiency by allowing continuous recording of overlapping shots. In simultaneous source acquisition, instead of firing one shot each time and waiting for its seismic response, several shots are fired with at close time intervals. In land acquisition, different phase-encoding schemes have been utilized to distinguish the signal from different Viboseis (Bagaini, 2006). In marine acquisition, simultaneous shooting relies on the randomization of the firing time delays. This is because random time delays would preserve the coherency of desired signal while perturbing the interference in common receiver, offset and midpoint domains (Stefani et al., 2007). The latter is important as it allows separation of simultaneous source data via a coherent-pass constraint (Abma et al., 2010).

Various techniques have been developed for deblending simultaneous source data. Methods that exploit the low-rank property of the unblended data are of special interest to this paper. Maraschini et al. (2012) utilized the SSA low-rank filter, or equivalently the Cadzow filter, in an iterative manner to suppress the incoherent interference in common offset domain. Cheng and Sacchi (2013) posed deblending as a rank constrained inverse problem and solved it via the gradient projection method. Further developments include relaxing the low-rank constraint to a nuclear norm constraint that leads to better properties of convergence (Cheng and Sacchi, 2014; Wason et al., 2014). One major concern of these rank-reduction based deblending methods is computational cost. Rapid rank reduction methods, such as Lanczos bidiagonalization (Gao et al., 2011) and randomized SVD (Oropeza and Sacchi, 2011) were applied to seismic data de-noising and reconstruction.

In this article, we develop a fast rank reduction algorithm based on random projection and QR decomposition. The algorithm improves the computational efficiency compared to the singular value decomposition method. We also discussed an iterative rank reduction framework for simultaneous source separation of 3D common receiver gathers. We tested the proposed method with a synthetic 3D VSP data set under different scenarios where we have varied rank and survey time.

THEORY

Self-simultaneous source acquisition

We provide a brief review of the self-simultaneous shooting, which is a special case of simultaneous source acquisition (Abma et al., 2013). One vessel keeps traveling and firing until it covers the whole survey area. The detectors are ocean bottom nodes. To save acquisition time, shots are fired with small random time delays that introduce interference. The firing time of the l-th source is defined by

$$t_l = t_{l-1} + \tau_l = \sum_{i=1}^{l} \tau_i \,, \tag{1}$$

where τ_i is the time delay for the i-th source. We assume the source locations can be binned in a regular grid. For a single receiver, we denote the data associated to sources in the spatial positions x_l, y_l by a 3D tensor or multilinear array $\boldsymbol{\mathcal{D}}$. The blended data are then represented as follows

$$b(t) = \sum_{l \in \mathbb{S}} \mathcal{D}(t - \tau_l, x_l, y_l) \,. \tag{2}$$

The blending process shifts each shot record according to the firing time delay (τ_l) and superimposes the shot records into a super shot gather. One can rewrite Equation (2) in its operator form as follows

$$\mathbf{b} = \Gamma \boldsymbol{\mathcal{D}}, \tag{3}$$

where the blending operator is designated by Γ. Equation (3) is an underdetermined linear system of equations where the data collected by one receiver $\mathbf{b}$ contain information from multiple shots. The adjoint operator Γ^* is the pseudo-deblending operator. The latter entails shifting the time delays back and splitting the blended observation to one common receiver gather (Mahdad et al., 2011). It is equivalent to the minimum norm solution to the blending system of equation. It can be easily demonstrated that the minimum norm cannot remove shot interference and therefore, extra constraints are needed for finding the solution of Equation (3).

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Rank-constrained inversion

We will write the desired unblended seismic data $\mathcal{D}$ in terms of its Fourier transform as follows

$$\mathcal{D}(t,x_l,y_l) = \int \tilde{\mathcal{D}}(\omega,x_l,y_l)e^{i\omega t}\,d\omega\,, \tag{4}$$

At a given monochromatic frequency, the spatial data of the ideal unblended common receiver gather in frequency-space domain $\tilde{\mathcal{D}}$ can be represented via a matrix $\mathbf{D}_\omega$. $\mathbf{D}_\omega$ is a low-rank matrix (Trickett, 2003; Cheng and Sacchi, 2014). In simultaneous source acquisition, the interference from the blended shots would increase the rank of the $\mathbf{D}_\omega$. Therefore, we propose to minimize the misfit between the blended estimation and the observed data subjecting to a low-rank constraint that is applied to the data in frequency-space domain

$$\min J = \|\mathbf{b} - \Gamma\mathcal{D}\|_F^2 \quad s.t.$$
$$\forall\omega : \mathbf{D}_\omega \in \mathbb{C}_k = \{\mathbf{D}_\omega : rank(\mathbf{D}_\omega) \le k\}\,. \tag{5}$$

The solution is acquired via the Gradient Projection method (Cai et al., 2010; Ma et al., 2011). The iterative rank-reduction deblending framework is summarized in algorithm (1). We consider to successively update the current estimate $\mathcal{D}^\nu$ in the opposite direction of the gradient as follows

$$\mathcal{X} = \mathcal{D}^\nu - \lambda\Gamma^*(\Gamma\mathcal{D}^\nu - \mathbf{b})\,. \tag{6}$$

λ denotes the step size. A new solution is found by projecting $\mathcal{X}$ to a set of low rank matrices in frequency-space domain

$$\mathcal{D}^{\nu+1} = \mathscr{P}[\mathcal{X}]\,, \tag{7}$$

where $\mathscr{P}$ denotes the projection operator. The algorithm is initialized with the pseudo-deblended data $\Gamma^*\mathbf{b}$ as it contains exactly the information of the unblended signal.

Algorithm 1 Iterative Rank Reduction Deblending Algorithm

Inputs:
- Blending operator Γ and its adjoint Γ^*
- Observed blended trace $\mathbf{b}$
- Stopping criterion ε
- Step size λ

Initialize:
- $\mathcal{D}^0 = \Gamma^*\mathbf{b}$; $\nu = 0$;

repeat
- $\mathcal{X} = \mathbf{d}^\nu - \lambda\Gamma^*(\Gamma\mathcal{D}^\nu - \mathbf{b})$
- $\nu = \nu + 1$
- $\mathcal{D}^\nu = \mathscr{P}[\mathcal{X}]$ (See Algorithm 2)

until $\|\mathbf{b} - \Gamma\mathbf{d}^\nu\|_2^2 < \varepsilon$

$\mathbf{d} = \mathbf{d}^\nu$

Fast low-rank projection via rQRd

The projection operator in equation (7) entails transforming $\mathcal{X}$ to the frequency-space domain. At a given frequency ω, we denote the spatial data of $\tilde{\mathcal{X}}$ as $\mathbf{X}_\omega$. We then perform matrix rank reduction on $\mathbf{X}_\omega$ and repeat the process for all frequencies before transforming back to time domain. Instead of implementing matrix rank reduction via the truncated SVD (tSVD), we propose a simple, fast method named randomized QR decomposition (rQRd). In tSVD, we keep only the k largest singular values while setting all other singular values to zero. The reduced-rank approximation is computed by reconstructing the matrix with the new set of singular values. In rQRd, we first project $\mathbf{X}_\omega$, by a set of p random normalized vectors given by the matrix $\mathbf{\Omega}$:

$$\underset{NS_x\times p}{\mathbf{M}} = \underset{NS_x\times NS_y}{\mathbf{X}_\omega}\,\underset{NS_y\times p}{\mathbf{\Omega}}\,, \tag{8}$$

where NS_x and NS_y are the total number of shots in x and y directions, respectively. Owing to the randomness, the vectors in matrix $\mathbf{M}$ are linearly independent. Since the unblended data are low rank, only a number of p random vectors will be required to span the full range of the desired signal (Halko et al., 2011). We compute the orthonormal basis $\mathbf{Q}$ with the economy-size QR decomposition as follows

$$\underset{NS_x\times p}{\mathbf{Q}}\,\underset{p\times p}{\mathbf{R}} = \underset{NS_x\times p}{\mathbf{M}}\,. \tag{9}$$

The low rank approximation is then acquired via

$$\hat{\mathbf{X}}_\omega = \mathbf{Q}\mathbf{Q}^H\mathbf{X}_\omega\,. \tag{10}$$

The random projection reduces the size of matrix before applying QR decomposition as $p \ll NS_y$. We are able to minimize the computations of the procedure by operating on the reduced-size matrix. The rQRd projection operator is summarized in algorithm (2).

Algorithm 2 Projection operator via rQRd $\mathscr{P}$:

Inputs:
- Updated estimation from gradient: $\mathcal{X}$
- The rank in RQRD: p

Initialize:
- $\tilde{\mathcal{X}} \leftarrow \mathcal{X}$ (transform to frequency domain)

for $\omega = \omega_{min} : \omega_{max}$ **do**
- $\mathbf{M} = \mathbf{X}_\omega\mathbf{\Omega}$ (random projection)
- $[\mathbf{Q},\mathbf{R}] = \mathrm{qr}[\mathbf{M}]$
- $\hat{\mathbf{X}}_\omega = \mathbf{Q}\mathbf{Q}^H\mathbf{X}_\omega$

end for

$\mathcal{D}^{\nu+1} \leftarrow \hat{\mathcal{X}}$ (transform back to time)

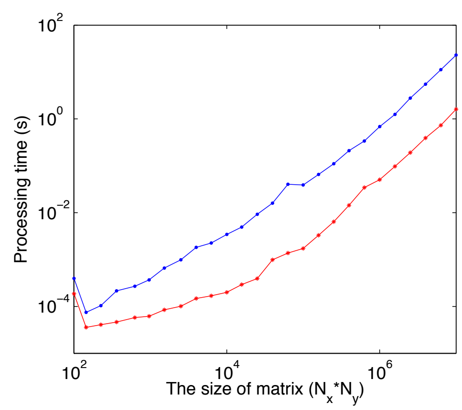

Figure 1: Processing time of rank reduction versus size of the matrix in logarithmic scale. The blue curve shows the processing time for the truncated SVD. The red curve corresponds to the rQRd method. In this example, we choose p equals to $3K$

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Figure (1) shows the comparison of processing time on a test where we perform matrix rank reduction using tSVD (blue) and rQRd (red). The rQRd algorithm is about 10 times faster than the conventional tSVD. As we will discuss later, the rank in rQRd, p, is a relaxation of the exact rank of a matrix (Chiron et al., 2014). Unlike tSVD which directly solves for the closest low-rank approximation of a given matrix, the rQRD does not constrain the rank as strongly. In rQRd we usually choose a rank (p) that is larger than the exact rank of the given matrix (K). The latter leads to good results when the singular values do not decay dramatically.

EXAMPLE

We use a synthetic 3D vertical seismic profile data set to mimic the process of simultaneous source acquisition. The data set contains 205 source lines with 205 sources on each line (O'Brien, 2010). The interval of each source position is 16.67m and the line spacing is also 16.67m. A total of 31 downhole detectors are deployed from 1350m to 1850m with 16.67m interval. The sources are blended using the self-simultaneous shooting technique. Under this scenario, the survey time ratio (STR), which is defined by the conventional survey time divide by the blended survey time (Berkhout, 2008), can be measured via

$$STR = \frac{\tau_0}{\bar{\tau}}, \tag{11}$$

where τ_0 denotes the regular firing time interval for the conventional seismic acquisition and $\bar{\tau}$ is the expected time delay for all the blended shots. For example, if the survey time ratio equals to 10, the expectation of randomly generated time delays will be 10% of one conventional shot record length. In other words, we are trying to save 90% of the total acquisition time of a conventional seismic survey. We measure the quality of deblending via

$$Q_S = 10log\frac{||\boldsymbol{\mathcal{D}}_{true}||_2^2}{||\boldsymbol{\mathcal{D}}_{true} - \boldsymbol{\mathcal{D}}_S||_2^2}, \tag{12}$$

where $\boldsymbol{\mathcal{D}}_{true}$ is the true synthetic data from a conventional common receiver gather and $\boldsymbol{\mathcal{D}}_S$ stands for the separated common receiver gather via iterative rank reduction.

We tested the effectiveness of the deblending algorithm in terms of different selections of rank for both tSVD and rQRd. The firing time delay is fixed in this experiment ($STR = 10$). As is show in Figure (2), the tSVD method (blue) presents the highest deblending quality only when selected rank is very close to the exact rank of data K. The rQRd method (red), on the other hand, can achieve reasonable results when the selected rank is in the range $p \in [1.5K, 5K]$. The test provides evidence that rank p in the rQRd algorithm is a relaxation of the desired rank K. In other words, we may not need accurate information about the rank of unblended data to perform the iterative reduced-rank deblending algorithm.

We also tested the deblending results under different survey time ratios and the rank p. At a given STR and p, we generated 50 realizations of firing time delay based on an uniform distribution. For each realization, we ran the deblending algorithm. If the quality of separation is higher than 20dBs, we consider the algorithm successfully separated the blended shots. In Figure (3), the white color means that in all 50 trials, the algorithm successfully removed the interferences. In contrast, the dark area indicates for those combinations of rank and STR, all the trials failed to improve the data quality factor to 20dB. The gray area is called the phase transition area in the field of compressive sensing (Donoho and Tanner, 2009), where the deblending algorithm has both succeeded and failed in deblending the data.

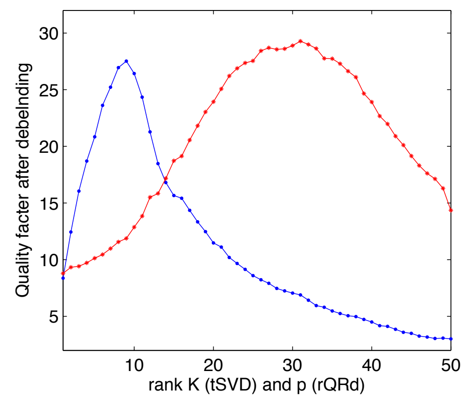

Figure 2: The quality of deblending versus rank K in tSVD and p for the rQRd. The blue curve shows the results utilizing the truncated SVD as the low rank projection operator while the red is the results corresponding to the rQRd method.

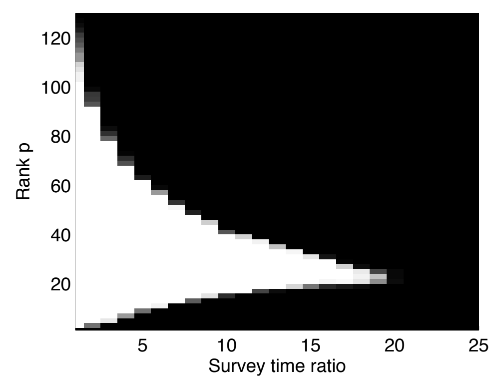

Figure 3: The probability map of simultaneous source separation with the proposed deblending method under different rank and survey time ratio. For small STRs, a broad range of ranks could be adopted to achieve successful separations. As the STR grows, we need precise knowledge of the optimum rank to correctly deblend the data. For this specific model and acquisition design, the algorithm performs poorly when STR is greater than 21.

Figure (4) shows the deblending result for the center shot and Figure (5) shows the result for the receiver at the center of the survey. The interferences from simultaneously fired shots are effectively suppressed. We improve the quality factor of the pseudo-deblended dataset to 32.5 dB. We find that deblending on 3D common receiver gathers usually lead to better results compared to 2D deblending methods. This is because by operating on the 3D gathers we introduce one extra degree of

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randomness with respect to the firing time.

CONCLUTION

We present an iterative rank-reduction algorithm for simultaneous source separation. The method operates on 3D common receiver gathers and relies on the randomization of the firing time delays. A cost function is defined by the blending system and a low-rank constraint. We implement matrix rank reduction via randomized QR decomposition. The latter significantly saves computation time when compared with classical rank-reduction via truncated SVD. In addition, the randomize QR decomposition does not require precise knowledge of the rank of the unknown solution. We tested the effectiveness of the deblending algorithm with a synthetic 3D VSP data set. We also tested the performance of the algorithm with different selections of rank and survey time. The algorithm also permits to reconstruct missing sources.

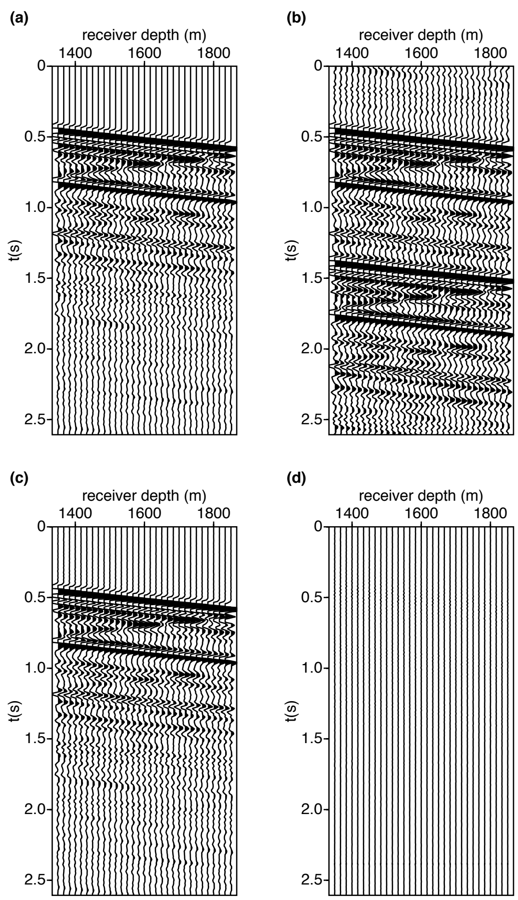

Figure 4: Results of simultaneous source separation in common shot domain. STR equals to 2. (a) The real unblended common shot gather. (b) Pseudo-deblended common shot gather. (c) Deblended common shot gather via the proposed algorithm. (d) Differences between (a) and (c). In this example, the signal-to-noise ratio after separation is 32.5dB.

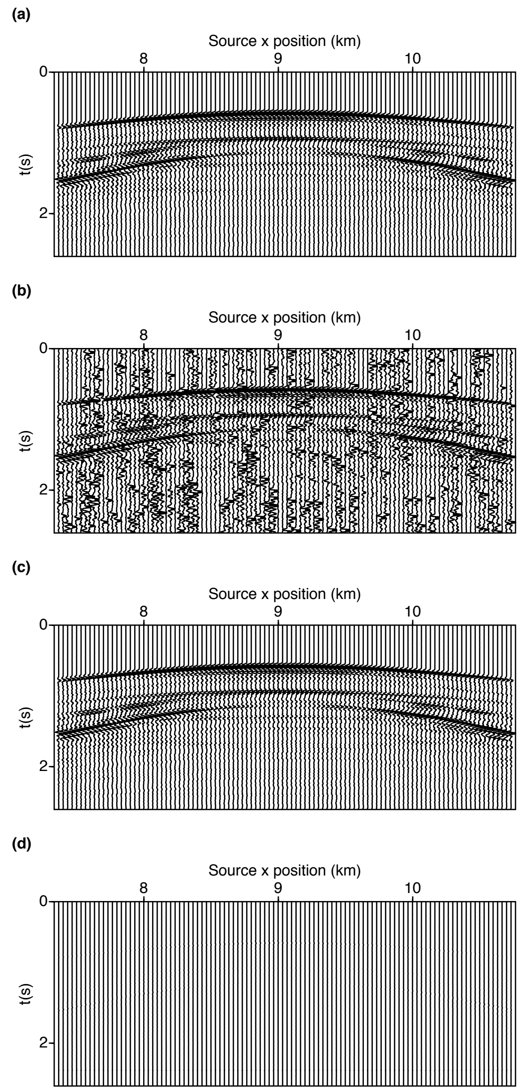

Figure 5: Results of simultaneous source separation in common receiver domain: (a) The real unblended common receiver gather. (b) Pseudo-deblended common receiver gather. (c) Deblended common receiver gather via the proposed algorithm. (d) Differences between (a) and (c).

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EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2015 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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