Mitigating artifacts in Projection Onto Convex Sets interpolation

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SUMMARY

Ringing artifacts are often observed above the water-bottom when applying POCS interpolation to marine datasets. The ringing is the result of hard thresholding creating sharp cut-offs in the frequency-wavenumber domain. Moreover, it is likely that the artifacts persist below the water-bottom but are masked by adjacent reflections. Modifying the type of thresholding used can mitigate artifacts, but often at the expense of introducing amplitude bias in the interpolated traces. We investigate several thresholding schemes and provide a generalized thresholding function that allows the style of thresholding to be controlled by a single parameter. We find that soft thresholding combined with a debiasing step provides improved results over hard thresholding in both marine and land datasets.

INTRODUCTION

Projection Onto Convex Sets (POCS) is an iterative optimization technique that has found broad application in seismic data processing including interpolation (Abma and Kabir, 2006; Stein et al., 2010), noise attenuation (Gao et al., 2013), and de-blending of simultaneous source data (Abma and Ross, 2013). It is an effective interpolation strategy that often out performs other methods in the presence of low *SNR* (Stanton et al., 2012).

Interpolation of marine data using POCS often results in a ringing effect above the water-bottom, an effect that is particularly noticeable in areas with large gaps between observed traces. We found that these artifacts are created by the application of the hard thresholding operator to the data in the frequency wavenumber domain. Soft thresholding can mitigate these artifacts, but this often leads to amplitude bias in the interpolated traces. Peyr (2010) detail a number of alternative thresholding strategies that offer a trade-off between these issues.

We investigate several thresholding strategies in POCS interpolation and arrive at a a generalized thresholding function that allows the style of thresholding to be controlled by a single parameter. Should the thresholding introduce amplitude bias for interpolated traces, we provide a simple amplitude compensation step using the median RMS amplitude of original traces. Synthetic and real data examples demonstrate the effectiveness of soft thresholding plus a debiasing step.

THEORY

One iteration of POCS interpolation generally consists of two projections. Firstly, a projection is made to keep only the highest amplitude Fourier coefficients (hard thresholding). A second projection resets observed traces to their original value. These two projections are illustrated in the equation

$$D^k(\omega,\vec{x}) = \alpha D^{obs}(\omega,\vec{x}) + (1-\alpha S)\mathcal{F}_{\vec{x}}^{-1} T^K \mathcal{F}_{\vec{x}} D^{k-1}(\omega,\vec{x}), \tag{1}$$

where $D^{k-1}(\omega,\vec{x})$ are the data for a given temporal frequency ω at iteration $k-1$, $\mathcal{F}_{\vec{x}}$ is the forward Fourier transform over the spatial axes, T^k is an iteration dependent thresholding operator acting in the frequency wavenumber domain, and $\mathcal{F}_{\vec{x}}^{-1}$ is the inverse Fourier transform over the spatial axes. Reinsertion of the original data is achieved by multiplying the thresholded data by the sampling operator, S, a diagonal matrix consisting of 1's in place of observations and 0's in place of missing traces, adding the result to the observed data $D^{obs}(\omega,\vec{x})$. Denoising of observed traces is achieved by averaging observed and estimated data using a scale factor α (Gao et al., 2013).

In some respects POCS interpolation is straightforward to implement because it requires only two operations. However, some care should be taken in designing the thresholding operator T^k. Typically T^k is defined to be hard thresholding

$$T^k_{hard} = \begin{cases} 1, & \text{if } |x| > \lambda^k \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where λ^k is a cut-off amplitude for iteration k. Alternatively, soft thresholding could be used

$$T^k_{soft} = \begin{cases} 1 - \frac{\lambda^k}{x}, & \text{if } |x| > \lambda^k \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

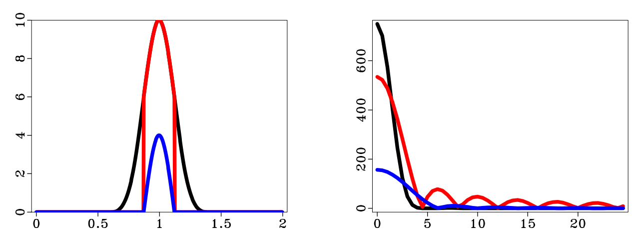

Figure 1: Three spectra (left) and their associated signals (right). The original signal (black) has a smoothly varying amplitude spectrum. Hard thresholding (red) produces significant ringing, while soft thresholding (blue) produces less ringing. For this example $\lambda = 6$.

In POCS interpolation the thresholding operator is sandwiched between the forward and inverse Fourier transform over the spatial axes. It is important to consider the Fourier response that will result from the action of the thresholding operator on the data. Figure 1 shows a synthetic signal (black) that has been thresholded by hard (red) and soft (blue) thresholding for a cut-off of $\lambda = 6$. Hard thresholding simply zeros amplitudes below the cut-off, resulting in a Gibbs effect, while soft thresholding subtracts the value of the cut-off from the data producing less ringing in its Fourier response. Introducing ringing artifacts into the interpolated data could be dangerous because these effects could mimic interpretable structure. Indeed, ”reconstruction methods should therefore be carefully designed to avoid spurious oscillations” (Donoho, 1995).

A drawback of iterative soft thresholding techniques is the introduction of amplitude bias. In denoising applications a ”debiasing” step is often carried out to compensate for the low amplitude of the estimated data. For seismic data interpolation debiasing is complicated by the fact that amplitudes cannot be estimated at all locations using original data.

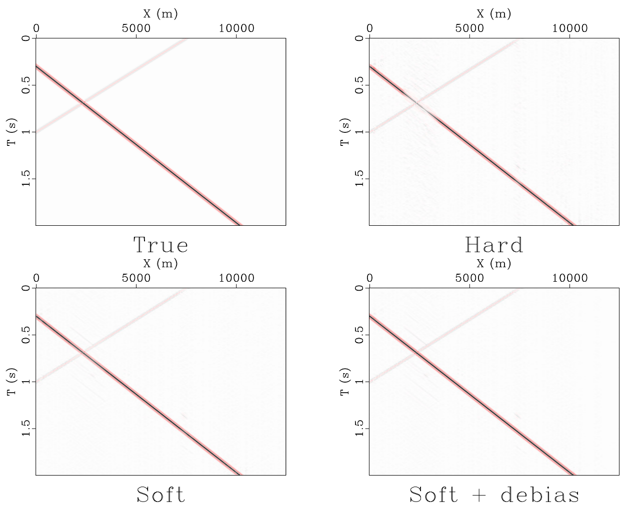

Figure 2: Results using various thresholding schemes for 200 iterations of POCS interpolation.

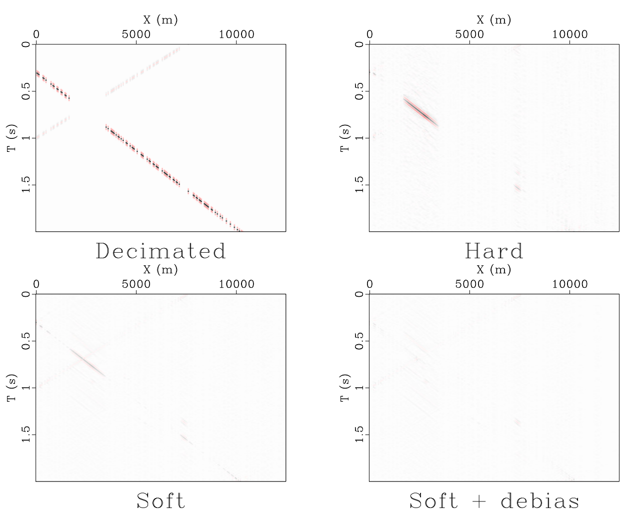

Figure 3: Difference panels using various thresholding schemes for 200 iterations of POCS interpolation.

Considering the ringing introduced by hard thresholding and the amplitude bias introduced by soft thresholding we might consider a thresholding scheme that lies somewhere between

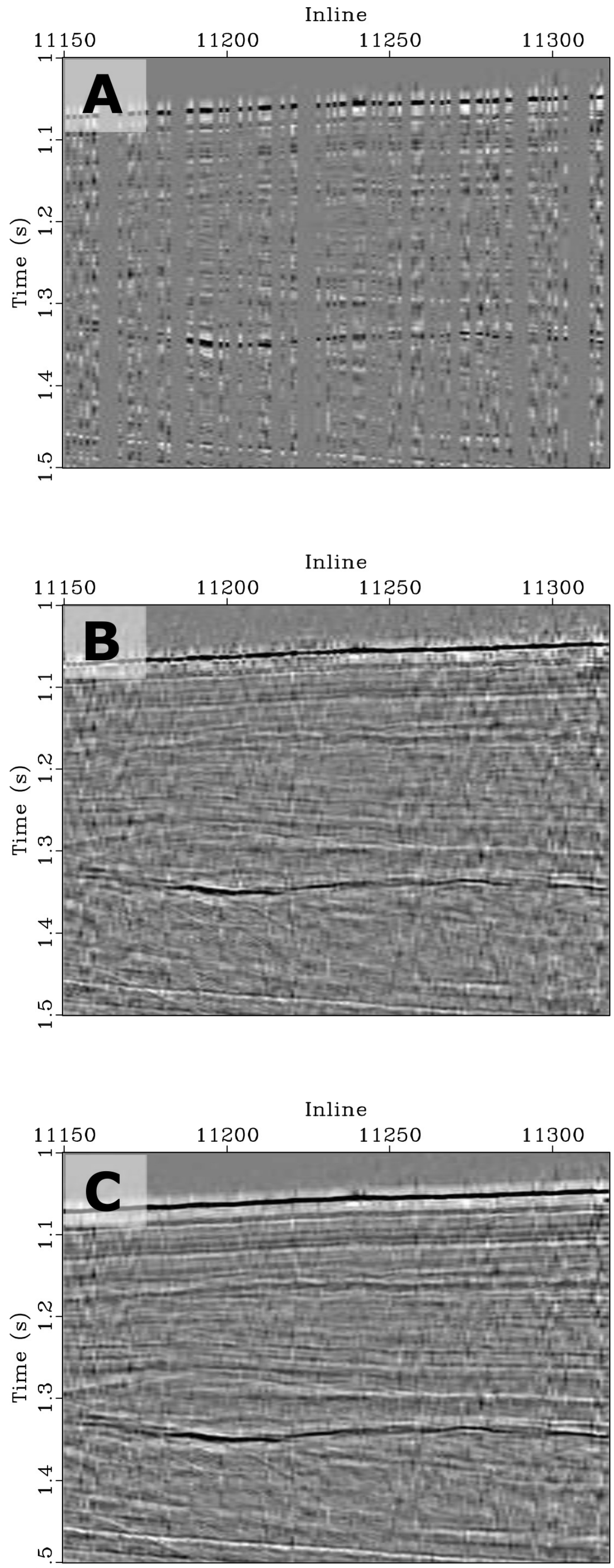

Figure 4: Cross line 54400 from a stack section acquired in the Gulf of Mexico with approximately 50% missing traces. (a) Original input data, (b) after POCS with hard thresholding and (c) after POCS with soft thresholding plus debiasing.

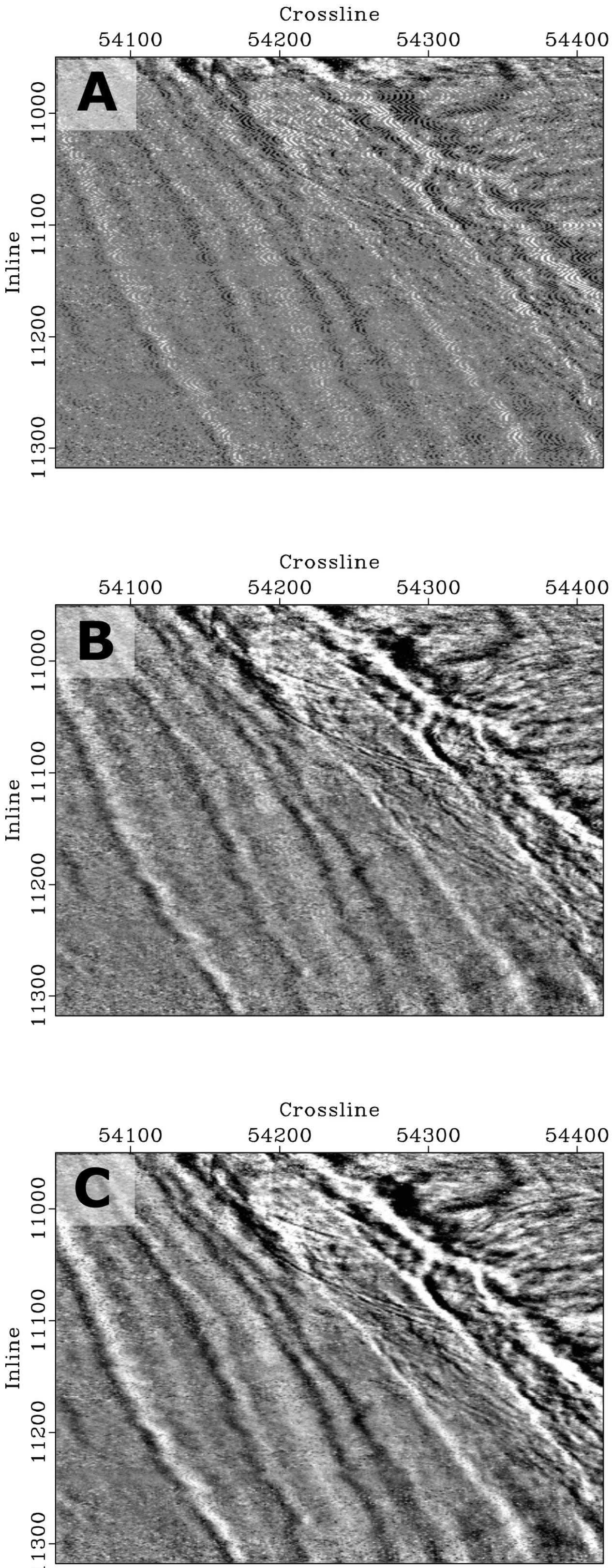

Figure 5: Time slice (1.6s) from a stack section acquired in the Gulf of Mexico with approximately 50% missing traces. (a) Original input data, (b) after POCS with hard thresholding and (c) after POCS with soft thresholding plus debiasing.

the two. Stein thresholding (Peyr, 2010) is defined by

$$T^k_{stein} = \begin{cases} 1-(\frac{\lambda^k}{x})^2, & \text{if } |x| > \lambda^k \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Which has the effect of tapering the edges of the cut-off amplitudes while lessoning the degree of amplitude bias. Peyr (2010) discuss a number of exotic thresholding schemes that attempt to find a balance between these two conflicting issues. We suggest a generalization of Stein thresholding

$$T^k_{general} = \begin{cases} 1-(\frac{\lambda^k}{x})^p, & \text{if } |x| > \lambda^k \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where the exponent p is a user defined parameter. A large value of p (e.g. p=100) is equivalent to hard thresholding, while a value of p=1 is equivalent to soft thresholding. Values falling between these two extremes provide more exotic thresholding schemes that can be used to achieve a balance between ringing artifacts and amplitude bias of the interpolated traces.

Choosing a thresholding exponent near to 1 (soft thresholding) can often result in interpolated traces that are lower in amplitude than the surrounding original traces. Because we interpolate the data in small multidimensional windows it is generally safe to scale the interpolated traces to the median of the RMS amplitude of the original traces. We formulate amplitude debiasing of the interpolated traces in three steps:

1. Compute the median RMS amplitude for all original traces
2. Scale the RMS amplitude of all interpolated traces to match this value
3. Smooth the RMS amplitude of all traces in each of the (up to four) spatial dimensions and reset the original traces to their input value.

Step 3 takes a POCS-like approach to amplitude debiasing and can be repeated a number of times to obtain spatially smooth RMS amplitudes for interpolated traces.

EXAMPLES

We first consider a 2D synthetic data example consisting of two dipping events shown in figures 2 and 3. 60% of the traces were randomly decimated prior to 200 iterations of POCS interpolation. Three strategies were tested: hard thresholding, soft thresholding, and soft thresholding plus an amplitude debiasing step. In figure 3 it is clear that soft thresholding outperforms hard thresholding for the large gap, while soft thresholding plus debiasing is able to interpolate the data almost perfectly.

Our next example considers a 3D stack section from a towed streamer survey acquired in the Gulf of Mexico. The input data have approximately 50% missing traces. Figure 4 shows a constant cross-line on input, after 100 iterations of POCS with hard thresholding, and after 100 iterations of POCS with soft thresholding plus debiasing. The result of soft thresholding

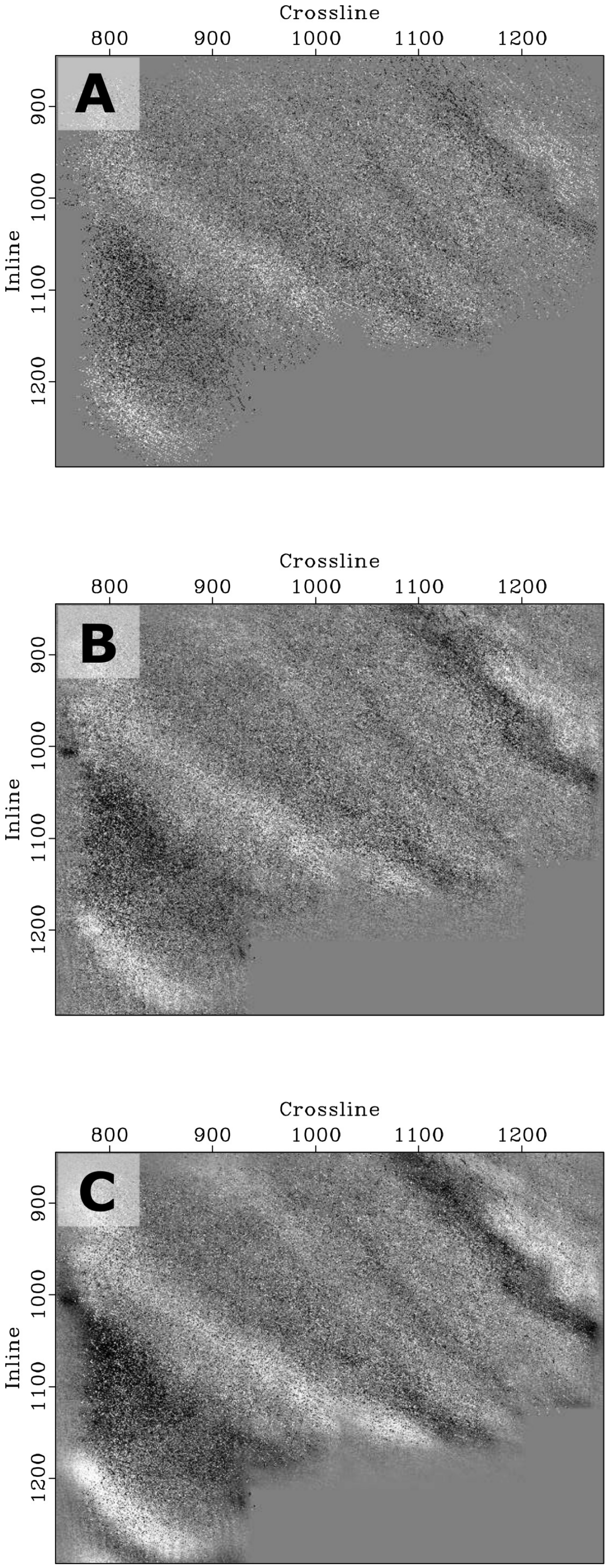

Figure 6: Time slice (2.35s) from a constant offset section acquired in the Eagle Ford Formation with approximately 50% missing traces. (a) Original input data, (b) after POCS with hard thresholding and (c) after POCS with soft thresholding plus debiasing.

plus debiasing is better than the result of hard thresholding in the region of the water-bottom as well as deeper in the section. Figure 5 shows results for a constant time-slice at 1.6s from the same dataset. It is again clear that soft thresholding plus debiasing improves the continuity of reflectors compared to hard thresholding.

Our last example is a constant offset section from the Eagle Ford Formation. Figure 6 shows a constant time-slice of 2.6s from the section after 100 iterations of POCS interpolation. The input section is missing approximately 50% of its traces. POCS interpolation with hard thresholding leads to a slightly noisy result, while POCS interpolation with soft thresholding plus debiasing improves the consistency of reflector amplitudes.

CONCLUSIONS

Ringing artifacts in POCS interpolation are created by sharp cut-offs in the thresholding operator acting in the frequency-wavenumber domain. The artifacts can be mitigated by changing the thresholding operator. We investigated several thresholding schemes and provide a generalized thresholding function that allows the style of thresholding to be controlled by a single parameter. Synthetic and real data examples demonstrate that soft thresholding combined with a debiasing step provides improved results over hard thresholding.

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