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SIMULTANEOUS SIGNAL AND NOISE MODELING VIA THE RADON TRANSFORM

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Summary

The decomposition and filtering of coherent noise is a long standing problem in exploration seismology. Low velocity events (air wave, surface waves or ground roll) tend to produce large energy signals that complicate the identification and enhancement of seismic reflections. This problem can be alleviated by filtering the offending low velocity signals using, for instance, $f-k$ filters. A major inconvenience with $f-k$ filters is that the signal (hyperbolas) and coherent noise (low velocity/linear event) may contain overlapping spectral components. Therefore, the filtering process will produce unavoidable signal distortions. A strategy to avoid the aforementioned problem entails the simultaneous modeling of the signal and noise components. An estimate of the coherent noise (modeled noise) is subtracted from the original data to obtain an estimate of a clean signal rich in hyperbolic events.

In this paper we discuss the implementation of the hybrid Radon transform as a mean to simultaneously model coherent noise and signal.

Introduction

The main step in noise attenuation is to transform the seismic data to a new domain where noise and signal can be easily separated. Radon transforms have been used to focus events with one of the following curvatures: linear, hyperbolic, and parabolic. Radon transforms have been efficiently applied for multiple suppression, ground roll filtering, and data interpolation. Reflections are usually approximated by hyperbolas or parabolas; whereas ground roll is considered a low velocity dispersive noise.

One way of modeling both the signal and low velocity noise is to adopt a combined operator that contains both hyperbolic and linear integration paths. This Radon transform was first introduced in Trad et. al (2001) and was denominated the *Hybrid Radon Transform.*

Theory

We assume that the seismic data (in this case a shot gather) is composed of 3 components: (i) events with hyperbolic moveout $\mathbf{d}_h$, (ii) low velocity linear events $\mathbf{d}_l$, and (iii) additive random noise $\mathbf{n}$. The recorded signal can be modeled as follows

$$\mathbf{d} = \mathbf{d}_h + \mathbf{d}_l + \mathbf{n}\,. \tag{1}$$

In addition we will model the hyperbolic and linear components in the data via Radon transforms. In this case we can write

$$\begin{aligned} \mathbf{d}_h &= \mathbf{L}_h\,\mathbf{m}_h \\ \mathbf{d}_l &= \mathbf{L}_l\,\mathbf{m}_l \end{aligned} \tag{2}$$

Where $\mathbf{L}_h$ and $\mathbf{L}_l$ define the Hyperbolic and Linear Radon Operators (Trad et. al, 2003), respectively. The Radon panels are designated by $\mathbf{m}_h$ (Hyperbolic) and $\mathbf{m}_l$ (Linear).

At this time we would like to point out that rather than attempting to directly estimating the signal components $\mathbf{d}_h$ and $\mathbf{d}_l$, we will invert their associated Radon panels $\mathbf{m}_h$ and $\mathbf{m}_l$. The system of equations we solve is given by:

$$\mathbf{d} = \mathbf{L}_h\,\mathbf{m}_h \,+\, \mathbf{L}_l\,\mathbf{m}_l \,+\, \mathbf{n}\,. \tag{3}$$

The latter is written in compact form as follows:

$$\mathbf{d} \,=\, \begin{pmatrix} \mathbf{L}_h & \mathbf{L}_l \end{pmatrix} \begin{pmatrix} \mathbf{m}_h \\ \mathbf{m}_l \end{pmatrix} \,+\, \mathbf{n}\,. \tag{4}$$

This is equivalent to solve a linear inverse problem of the form $\mathbf{d} \,=\, \mathbf{L}\,\mathbf{m} + \mathbf{n}$, where now $\mathbf{L}$ is an augmented Radon operator and $\mathbf{m}$ is the combined vector of Linear and Hyperbolic Radon domain parameters. The problem is solved using the Bayesian framework (Ulrych et al., 2001). In this case we assume that both observational errors and model space parameters are normally distributed with known data and model covariance matrices (the covariance matrices are bootstrapped from the data, see for instance, Trad et. al, (2003)). The Radon panels can be estimated by minimizing the cost:

$$J \,=\, \frac{1}{2}\,(\mathbf{Lm} - \mathbf{d})^T\,\mathbf{C}_n^{-1}\,(\mathbf{Lm} - \mathbf{d}) + \frac{1}{2}\,\mathbf{m}^T\,\mathbf{C}_m^{-1}\,\mathbf{m}\,. \tag{5}$$

Minimizing the last equation with respect to the unknown vector of parameters leads to

$$\hat{\mathbf{m}} \,=\, (\mathbf{L}^T\,\mathbf{C}_n^{-1}\,\mathbf{L} \,+\, \mathbf{C}_m^{-1})^{-1}\,\mathbf{L}^T\,\mathbf{C}_d^{-1}\,\mathbf{d}\,. \tag{6}$$

The following mathematical identity (with $\mathbf{B}$ and $\mathbf{A}$ positive definite matrices)

$$\mathbf{B}\,\mathbf{L}^T(\mathbf{A}^{-1} + \mathbf{L}\,\mathbf{B}\,\mathbf{L}^T)^{-1} = (\mathbf{B}^{-1} + \mathbf{L}^T\,\mathbf{A}\,\mathbf{L})^{-1}\,\mathbf{L}^T\,\mathbf{A} \tag{7}$$

permits to transform equation (6) into a form that resembles a minimum norm solution:

$$\hat{\mathbf{m}} \,=\, \mathbf{C}_m\,\mathbf{L}^T\,(\mathbf{C}_n \,+\, \mathbf{L}\mathbf{C}_m\,\mathbf{L}^T)^{-1}\,\mathbf{d} \tag{8}$$

Let us assume that the model covariance matrix can be written in the following general form

$$\mathbf{C}_m \,=\, \begin{pmatrix} \mathbf{C}_{hh} & \mathbf{C}_{hl} \\ \mathbf{C}_{lh} & \mathbf{C}_{ll} \end{pmatrix} \tag{9}$$

where $\mathbf{C}_{hh}$ and $\mathbf{C}_{hh}$ are the covariance matrices for the hyperbolic and linear Radon panels, respectively. Moreover, we consider the particular case where we assume no correlation between the linear and hyperbolic Radon panels, we set $\mathbf{C}_{lh} = \mathbf{C}_{hl} = \mathbf{0}$. Under these assumptions, the solution given by equation (8) reduces to the following form:

$$\begin{pmatrix} \hat{\mathbf{m}}_h \\ \hat{\mathbf{m}}_l \end{pmatrix} \,=\, \begin{pmatrix} \mathbf{C}_{hh} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{ll} \end{pmatrix} \begin{pmatrix} \mathbf{L}_h^T \\ \mathbf{L}_l^T \end{pmatrix} \left(\mathbf{L}_h\,\mathbf{C}_{hh}\,\mathbf{L}_h^T + \mathbf{L}_l\,\mathbf{C}_{ll}\,\mathbf{L}_l^T + \mathbf{C}_n \right)^{-1}\,\mathbf{d} \tag{10}$$

We can use expression (10) to find an estimator of the linear noise $\hat{\mathbf{d}}_l = \mathbf{L}_l\,\hat{\mathbf{m}}_l$

$$\hat{\mathbf{d}}_l \,=\, \mathbf{L}_l\,\mathbf{C}_{ll}\,\mathbf{L}_l^T\,(\mathbf{L}_h\,\mathbf{C}_{hh}\,\mathbf{L}_h^T + \mathbf{L}_l\,\mathbf{C}_{ll}\,\mathbf{L}_l^T + \mathbf{C}_n)^{-1}\,\mathbf{d}\,. \tag{11}$$

The above problem is solved using conjugate gradients. In our algorithm we split the problem into parts. First we use the method of conjugate gradients to solve

$$(\mathbf{L}_h\,\mathbf{C}_{hh}\,\mathbf{L}_h^T + \mathbf{L}_l\,\mathbf{C}_{ll}\,\mathbf{L}_l^T + \mathbf{C}_d)\,\mathbf{z} \,=\, \mathbf{d}\,.$$

then, we use $\mathbf{z}$ to compute the estimator of $\mathbf{d}_l$,

$$\hat{\mathbf{d}}_l \;=\; \mathbf{L}_l\,\mathbf{C}_{ll}\,\mathbf{L}_l^T\,\mathbf{z}$$

The last expression is utilized to estimate the low velocity noise from the seismic gather. It is interesting to notice that the estimator given in equation (11) can be interpreted as a *generalized notch filter*. In particular we define the following data space covariance matrices:

$$\begin{aligned}\mathbf{C}^d_{ll} &= \mathbf{L}_l\,\mathbf{C}_{ll}\,\mathbf{L}_l^T \\ \mathbf{C}^d_{hh} &= \mathbf{L}_h\,\mathbf{C}_{hh}\,\mathbf{L}_h^T\end{aligned} \tag{12}$$

The estimator of the linear noise (equation (11)) becomes

$$\hat{\mathbf{d}}_l \;=\; \mathbf{C}^d_{ll}\,(\mathbf{C}^d_{hh} + \mathbf{C}^d_{ll} + \mathbf{C}_n)^{-1}\,\mathbf{d}\,. \tag{13}$$

The last expression resembles the expression of a notch filter utilized to cancel random noise and one coherent signal component (Soubaras, 1994). At this point it is important to notice that the computation of the covariance matrices of the signal in data space (hyperbolic and linear events) is not an easy task. Approximations based on smoothing arguments can be used to define data covariance matrices or equivalent model space weights. The procedure outlined above permits a more natural representation of the signal covariance matrices in terms of the covariance of the signal in the transformed domain. The latter is a point to further study for the simultaneous modeling of primaries and multiples and the subsequent adaptive subtraction of the estimated multiple model (Luo et al., 2003).

Example

Figure 1 illustrates the Radon panels obtained with the Hybrid Radon Transform for a shot gather contaminated with low velocity coherent noise. In general, the Radon panels do not need to be computed since we can always use a direct implementation of equation (13). Figure 2a shows the original data. Figure 2b is the data predicted using simultaneously the Linear and Hyperbolic components. The error panel is portrayed in Figure 2c. This is also an estimate of the incoherent noise component $\mathbf{n}$ in equation (1). The reconstruction of the linear events (via expression (13)) is presented in Figure 2d. The filtered data, the original gather minus the estimated low velocity noise, is portrayed in Figure 2e. In this example the covariance matrices in model space where computed using weights derived from an initial least squares solution as discussed in Trad et al. (2003).

Conclusions

We have described an algorithm to remove low velocity noise based on the the hybrid Radon transform. We have also outlined a procedure to compute data space covariance operators for both linear and hyperbolic events (equation (12)). The model covariance matrix in the transformed domain can be easily computed as demonstrated by Trad et. al (2003). The proposed method can also be extended for designing data covariance matrices for the simultaneous inversion of multiples and primaries (Luo, 2003).

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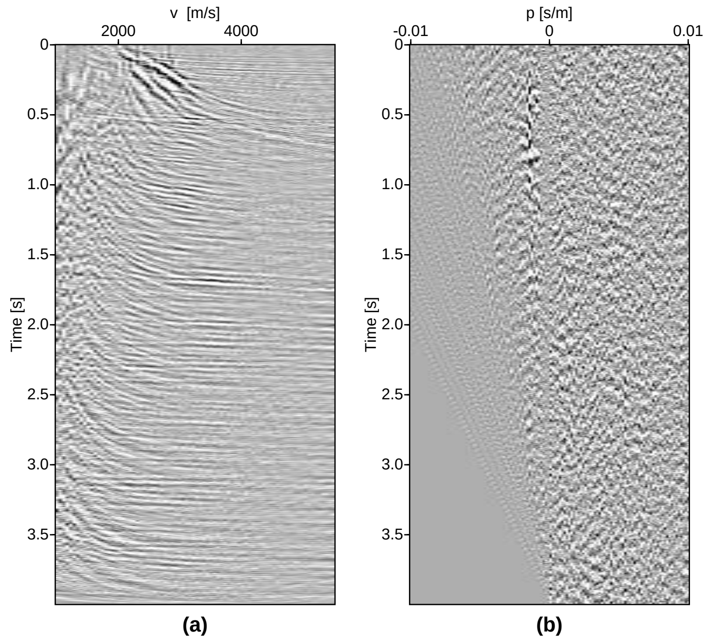

Figure 1: Hybrid Radon Trasnform. (a) Hyperbolic events $\hat{\mathbf{m}}_h$. (b) Linear events $\hat{\mathbf{m}}_l$.

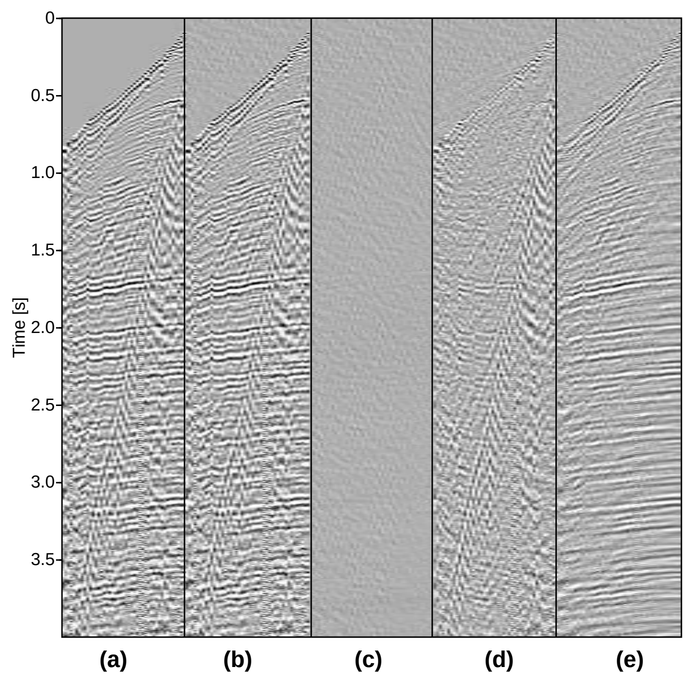

Figure 2: (a) Input shot gather $\mathbf{d}$. (b) Predicted shot gather using both linear and hyperbolic events $\hat{\mathbf{d}} = \mathbf{L}_l\hat{\mathbf{m}}_l + \mathbf{L}_h\hat{\mathbf{m}}_h$. (c) Error panel $\mathbf{d} - \hat{\mathbf{d}}$. (d) Prediction of linear events $\hat{\mathbf{d}}_l = \mathbf{L}_l\,\hat{\mathbf{m}}_l$. (e) Hyperbolas predicted by subtracting the linear events from the original data $\hat{\mathbf{d}}_h = \mathbf{d} - \hat{\mathbf{d}}_l$