Robust AVP estimation using least-squares wave-equation migration

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Summary

Wave-equation migration is well known for its capability to handle structural imaging in complex geological settings. In recent years, imaging principles have been developed that can be used to extract angle dependent reflectivity from pre-stack seismic data. The angle or offset ray parameter dependent reflectivity (AVA or AVP) is estimated from the downward continued wavefield. We propose a least-squares (LS) approach to wave-equation migration that enables us to generate high quality common image gathers (CIGs) that exhibit amplitude-variations-with-ray-parameter (AVP). In a previous contribution, we have demonstrated, with the aid of the Marmousi dataset, that least-squares imaging with a smoothing constraint on the CIGs reduces kinematic artifacts in the CIGs. In this paper, we further explore the effect of the smoothing regularization term for imaging/inversion of incomplete and noisy data. A relatively simple example allows us to better assess the performance and appropriateness of the LS smoothing regularization in terms of AVP preservation. The results are promising and suggest that least-squares migration, although computationally expensive, is beneficial for the computation of high-quality AVP estimates.

Introduction

In recent years, increasing attention has been given to wave-equation migration imaging conditions that attempt to retrieve amplitude variations versus angle of incidence (AVA) or offset ray-parameter (AVP) (e.g., Stolt and Weglein, 1985; de Bruin et al., 1990; Prucha et al., 1999; Wapenaar et al., 1999; Mosher and Foster, 2000 and Sava et al., 2001). This paper focuses on the ray parameter imaging condition as described, for instance, in Sava et al. (2001). As opposed to migration in the tau-p domain (Ottolini, 1984), ray-parameter imaging extracts constant (half)offset ray parameter gathers from the downward continued wavefield thereby relaxing the restriction to horizontally layered media. We combine extended double-square-root (DSR) propagators (e.g., Clayton and Stolt, 1981; Gazdag and Sguazzero, 1984 and Stoffa et al., 1990) with the ray parameter imaging technique in a least-squares (LS) optimization algorithm with a smoothness constraint on the ray parameter common image gathers (CIGs). This idea has similarities to the algorithm proposed by Duquet et al. (2000) who use smoothing constraints to mitigate migration artifacts in Kirchhoff migration. It is important to further investigate how truthfully the constrained LS migration can estimate AVP characteristic and how missing data and noise influence the inversion success.

Theory

We employ modeling and migration wave-equation operators to invert the linear system:

$$\mathbf{d} = \mathbf{L}\,\mathbf{m} + \mathbf{n}, \tag{1}$$

where **d** is the usually incomplete and irregularly sampled data vector in half-offset *h*, midpoint *y* and frequency space. The term **n** represents additive noise. The model vector **m** is the ray parameter *p* dependent reflectivity at the midpoint position *y* and depth *z*. The modeling operator **L** is a combination of the adjoint of the ray parameter imaging operator described in Sava et al. (2001) and the double-square-root (DSR) upward wavefield propagator. Depending on the complexity of the underlying migration velocity field the DSR propagator is extended by a split-step operator and can also be applied in a multiple-reference velocity mode (Gazdag and Sguazzero, 1984; Stoffa et al., 1990).

The following cost function is iteratively minimized by means of a conjugate gradients (CG) algorithm:

$$F(\mathbf{m}) = \|\mathbf{W}(\mathbf{d} - \mathbf{L}\mathbf{m})\|^2 + \lambda^2 \|\partial_p \mathbf{m}\|^2, \tag{2}$$

where **W** is a diagonal weighting operator with zero weights for dead traces and non-zero weights for live traces. The CG algorithm is a semi-iterative scheme where the minimum of F(**m**) is sought in successive steps where the adjoint or migration operator **L'** and the modeling operator **L** are iteratively applied. Besides the data-misfit term, we have added a regularization term that penalizes roughness (rapid amplitude changes) along the ray-parameter *p*. Roughness is attributed to missing data, noise and numerical operator artifacts. The tradeoff parameter λ determines the amount of smoothing. This regularization approach has proven successful in a kinematic sense when applied to the Marmousi dataset (Kuehl and Sacchi, 2001).

The inverted ray parameter model gathers **m** can be converted to AVA gathers by means of the relationship:

$$\theta = \arcsin\left(\frac{vp}{2\cos\phi}\right), \tag{3}$$

where θ is the angle of incidence on a (assumed) locally plane reflector element, *v* is the migration velocity directly above the reflector and ϕ is the local reflector dip. The local reflector dip is obtained from the structural image.

Sava et al. (2001) point out that the AVP preservation is improved when an approximation of the ray parameter imaging Jacobian $\mathbf{J} \approx \mathbf{L'L}$ is considered when, instead of full LS migration, only the migration operator is applied. Furthermore, the Jacobian **J** can be used to precondition the least-squares inversion.

Example

We generated a dataset based on a simple horizontally layered model with a maximum depth of 1400m. The data were modeled with a ray-tracing code that does not account for transmission loss. In this case, this simplification is desired, since we wish to investigate the performance of the constrained LS inversion and not "true-amplitude" aspects of wavefield propagators in general. It is clear that in high-contrast media with significant transmission effects the amplitude fidelity of one-way wavefield propagators breaks down.

In this test, the angle dependent reflection coefficients at the three interfaces were modeled using Shuey's approximation of the Zoeppritz equations (Shuey, 1985). Of course, any other expressions for the reflection coefficient (e.g., acoustic reflection coefficients) could have been used as well. In any case, we consider only sub-critical reflections. The data consist of 100 midpoints and 64 offsets ranging in nominal half-offset from 0 to 660 m. To test the effect of dead data traces 50% of the data were randomly set to zero (Figure 2 a)). White noise with a signal-to-noise ratio of 10 was added. The ray parameter CIG of the migrated data is shown in Figure 1 a). The maximum half-offset ray parameter for the first reflector is 794 μs/m. Figure 1 b) and 1 c) show the CIGs after 4 and 9 iterations of the CG algorithm, respectively. The data misfit as a function of iteration number is shown in Figure 2 b). The least-squares migrated CIGs are cleaner and finite aperture artifacts are mitigated. We used a smoothing parameter of λ=0.01 and a ray parameter sampling step of Δp=16 μs/m.

In order to compare the AVP with the true AVP we picked the amplitudes on the CIGs. The results are depicted in the Figures 2 c), 3 a), and 3 b). For comparison, the true AVP is also shown. The least-squares migrated AVP CIGs come closest to the true AVP. They are continuous, show the correct slope, and they are of the correct relative strength to each other. Inevitable finite offset-wavenumber effects are still apparent at large ray parameters. We have also applied the inverse of the imaging Jacobian **J** to the migrated CIG (Figure 2 c)). In the case of a horizontally layered medium the Jacobian simplifies to $\mathbf{J}=v/(2\cos\theta)$. This factor was also derived by Wapenaar et al. (1999). To illustrate the effect of conventional migration plus averaging we smoothed the migrated AVP in Figure 3 c) using a 6 point moving average filter.

Conclusions

Constrained least-squares wave-equation migration for AVP inversion allows for the generation of high quality amplitude-variation-with-ray-parameter estimates, even when noise and incompleteness compromise the data. Extended DSR modeling/migration with ray parameter imaging is kinematically not restricted to horizontally invariant media. However, the development and testing of wave-equation propagators in complex media is a topic of ongoing work. For instance, Wapenaar et al. (1999) demonstrate how to account for reflection- and propagation related fine layering effects that cause an apparent AVA/AVP. Future research will show how well least-squares wave-equation migration can help to produce reliable AVP estimates in more intricate media.

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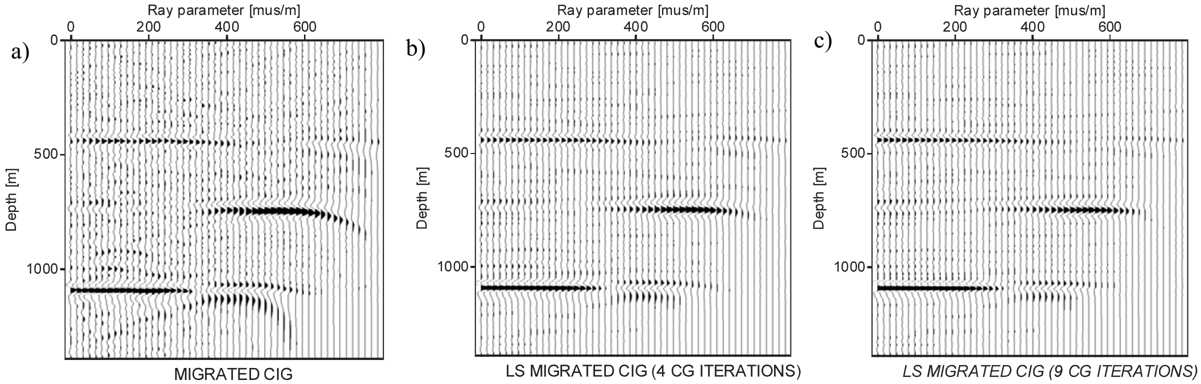

Figure 1: a) Migrated CIG. b) LS migrated CIG after 4 CG iterations. c) LS migrated CIG after 9 CG iterations.

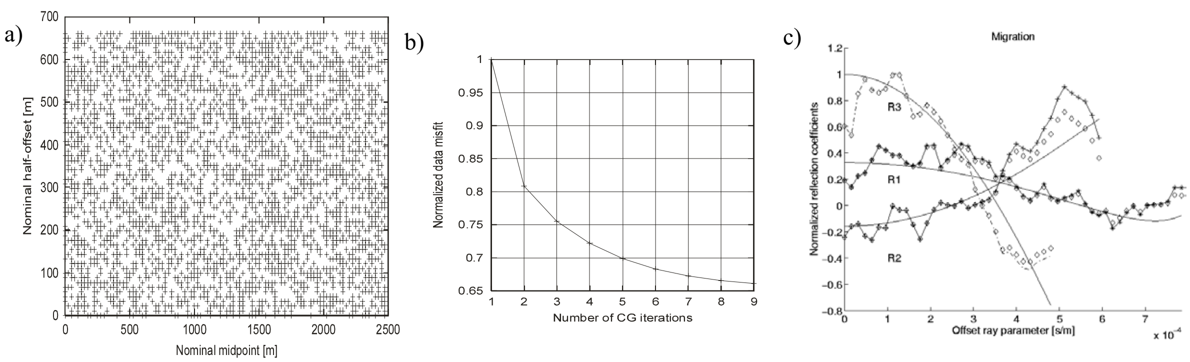

Figure 2: a) Map indicating the distribution of live traces (+). b) Normalized data misfit. c) Picked AVP curves from the CIG in Figure 1 a). The diamonds are the AVP after application of the ray parameter imaging Jacobian.

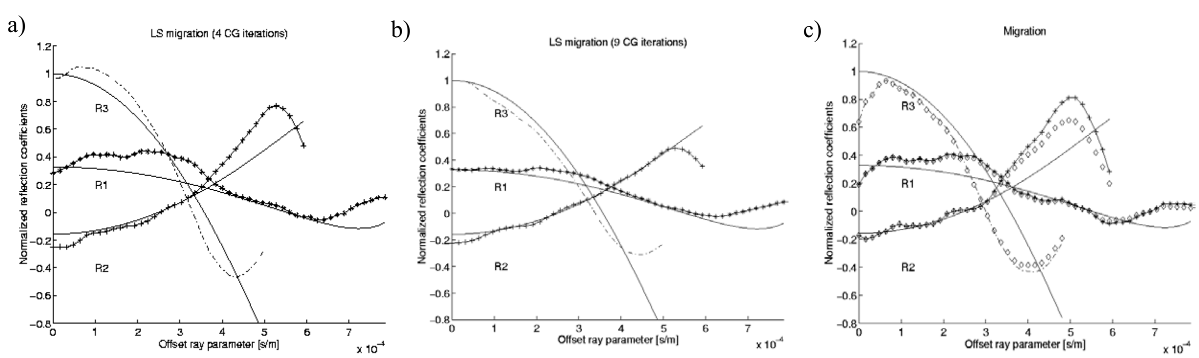

Figure 3: a) AVP curves from the CIG in Figure 1 b). b) AVP curves from the CIG in Figure 1 c). AVP curves from Figure 2 c) after application of a 6 point moving average filter.