A dual domain algorithm for minimum nuclear norm deblending

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SUMMARY

This paper illustrates an inversion scheme for separating simultaneous sources. The proposed algorithm assumes that an ideal 3D common receiver gather can be represented via a low rank matrix in the frequency-space domain. We propose a dual-space algorithm that minimizes the misfit between the observed blended data and predicted blended data in $t-x-y$ subject to a nuclear norm constraint that is applied to the data in the $\omega-x-y$ domain. The algorithm is illustrated with a synthetic 3D vertical seismic profile (VSP) data set.

INTRODUCTION

Simultaneous source acquisition, or blended acquisition, is implemented by firing several sources in a short period of time. The responses are recorded by a set of receivers that contain a considerable amount of contamination produced by overlapping sources. The method of blended acquisition permits to save acquisition costs at the expense of introducing extra processing steps to unmix the recorded data.

In marine data acquisition, blending techniques are often based on the randomization of time delays among air guns. The key is to make interferences appear random in common receiver gathers (Stefani et al., 2007; Hampson et al., 2008) and therefore, treat the problem via denoising methods. Coherent pass operators such as $f-k$ filters, median filters (Huo et al., 2009), $f-x$ SSA filter (Maraschini et al., 2012) and prediction subtraction techniques (Spitz et al., 2011; Mahdad et al., 2011) have been adopted to annihilate the interferences. The unblended data can also be estimated by sparsity promoting inversion techniques that operate in different domains (Abma et al., 2010; Moore, 2010; Mansour et al., 2012).

In a previous article (Cheng and Sacchi, 2013), we proposed a simultaneous source separation algorithm that is based on a rank-reduction technique. In particular, we developed rank-reduction schemes that operates on Hankel matrices that are formed from spatial data in the $f-x$ domain. A projection operator was utilized to recover the ideal deblended data that was assumed to be low-rank when embedded in a Hankel matrix. Recent work in the field of matrix completion expands methodologies for reduced-rank filtering by introducing algorithms that minimize the nuclear norm (sum of singular values) of a matrix (Fazel, 2002; Candes and Recht, 2009; Cai et al., 2010; Ma et al., 2011). Nuclear norm minimization was also adopted by Kreimer et al. (2013) to reconstruct 5D volumes. However, Kreimer et al. (2013) applied nuclear norm minimization on tensors rather than matrices.

In this paper, we replace the rank constraint utilized in Cheng and Sacchi (2013) by a regularization term based on nuclear norm. We also discussed a dual domain algorithm where deblending is carried out in the time domain with rank constraints that are applied to data in the frequency domain. The nuclear norm minimization problem can be solved via the classic gradient descent method in conjunction with a rank projection operator that is implemented via soft singular value thresholding (Cai et al., 2010). We apply our technique to a synthetic 3D VSP data set.

THEORY

We will consider a regular distribution of sources in $x-y$ (Figure (1)). The data associated to sources in the spatial positions x_l, y_l is designated by $d(t, x_l, y_l)$. We assume a total number of N_s sources.The blended data are represented as follows

$$b(t) = \sum_{l \in \mathbb{S}} d(t-\tau_l, x_l, y_l), \qquad l = 1:N_s \tag{1}$$

the latter can be written in operator form as follows

$$\mathbf{b} = \Gamma \mathbf{d} \tag{2}$$

where $\mathbf{b}$ is the blended data collected by one receiver and $\mathbf{d}$ indicates desired unblended data in the time domain. The blending operator is represented by Γ. We will remind the reader that the desired data $\mathbf{d}$ can be written in term of its Fourier transform as follows

$$d(t, x_l, y_l) = \int D(\omega, x_l, y_l) e^{i\omega t}\, d\omega, \quad l = 1:N_s. \tag{3}$$

If we consider a regular distribution of sources in the $x-y$ plane, one can express $D(\omega, x_l, y_l)$ in terms of a matrix $\mathbf{D}(\omega)$ of size $NS_x \times NS_y$, where the total number of sources is given by $NS = NS_x \times NS_y$. We will assume that the matrix $\mathbf{D}(\omega)$ is low-rank and therefore, we will pose deblending as a low-rank matrix completion problem. The deblended data are estimated by minimizing the following cost function

$$J = \|\mathbf{b} - \Gamma \mathbf{d}\|_2^2 + \mu \sum_{\omega} \|\mathbf{D}(\omega)\|_*. \tag{4}$$

In the above notation $\|\mathbf{D}(\omega)\|_*$ is the nuclear norm of the matrix $\mathbf{D}(\omega)$ which is given by the following expression

$$\|\mathbf{D}(\omega)\|_* = \sum_i S_i(\omega)$$

where $S_k(\omega)$ indicates the ith singular value of $\mathbf{D}(\omega)$. The singular values are positive. Consequently, to minimize the nuclear norm is equivalent to minimize the l_1 norm of the singular values. By minimizing the nuclear norm, one attempts to sparsify the spectrum of singular values.

Deblending via nuclear norm minimization

The classic gradient descent algorithm followed by a low-rank

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projection operator (Cai et al., 2010; Ma et al., 2011) is used to minimize the cost function of our problem

$$\begin{aligned} \mathbf{x} &= \mathbf{d}^{k-1} - \tau\Gamma^*(\Gamma\mathbf{d}^{k-1} - \mathbf{b}) \\ \mathbf{d}^k &= \mathscr{P}_{\tau\mu}[\mathbf{x}] \end{aligned} \quad (5)$$

where τ is the step size of the gradient descent algorithm. We remind the reader that $\mathbf{d}^k$ and, therefore $\mathbf{x}$ are deblended data at iteration k in the $t-x-y$ domain. However, the rank-reduction constraint $\mathscr{P}_{\tau\mu}$ must be applied in the $\omega-x-y$ domain. Consequently, we first transform the data $\mathbf{x}$ in $t-x-y$ domain to $\mathbf{X}(\omega)$ in $f-x-y$ domain. Then, for each temporal frequency ω, we run rank reduction via singular value soft thresholding. The latter is equivalent to compute the singular value decomposition (SVD) of $\mathbf{X}(\omega)$ and then, reconstruct the data with a new set of singular values that are given by $\hat{S}_k(\omega) = max[S_k(\omega) - \tau\mu, 0]$. In other words, if we denote $\mathbf{U}(\omega)$ and $\mathbf{V}(\omega)$ the matrices of singular vectors of $\mathbf{X}(\omega)$, the approximated low-rank matrix is given by $\hat{\mathbf{X}}(\omega) = \mathbf{U}(\omega)\hat{\mathbf{S}}(\omega)\mathbf{V}^*(\omega)$. $\hat{\mathbf{S}}(\omega)$ is the diagonal matrix of singular values after soft thresholding. To continue with the algorithm, the rank-reduced data in the $\omega-x-y$ domain, $\hat{\mathbf{X}}(\omega)$, is transformed back to the time domain to obtain a new estimator of the deblended data $\mathbf{d}^k$. The proposed gradient descent algorithm operates in time domain. However, the rank constraints are applied in the $\omega-x-y$ domain.

The algorithm is initialized with the pseudo-deblended data $\Gamma^*\mathbf{b}$. Convergence is guaranteed by adopting a step size, $\tau < 2/\sigma_{max}$, where σ_{max} is the maximum eigenvalue of the operator $\Gamma^*\Gamma$ (Ma et al., 2011). The proposed method adopts nonsummable diminishing steps that decrease according to $1/\sqrt{k}$ (Boyd and Mutapcic, 2007). At early iterations, the projection operator applies harsh rank-reduction filters to eliminate strong crosstalk. As iterations progress, we gradually relax the threshold to allow modeling details that require a representation in terms of a larger number of singular values. This is analogous to setting a threshold schedule in projection-onto-convex sets regularization and deblending methods (Abma et al., 2010). The procedure is provided in Algorithm 1 and Algorithm 2.

Algorithm 1 Dual Domain Minimum Nuclear Norm Deblending Algorithm

Inputs:
- Blending operator Γ and its adjoint Γ^*
- Observed blended trace $\mathbf{b}$
- Trade-off parameter μ
- Stopping criterion ε
- Initial step size τ_0

Initialize:
- $\mathbf{d}^0 = \Gamma^*\mathbf{b}$; $k = 1$;

repeat
- $\tau = \tau_0/\sqrt{k}$
- $\mathbf{x} = \mathbf{d}^{k-1} - \tau\Gamma^*(\Gamma\mathbf{d}^{k-1} - \mathbf{b})$
- $\mathbf{d}^k = \mathscr{P}_{\tau\mu}[\mathbf{x}]$ (See Algorithm 2)
- $k = k+1$

until $\|\mathbf{b} - \Gamma\mathbf{d}^k\|_2^2 < \varepsilon$

$\mathbf{d} = \mathbf{d}^k$

Algorithm 2 Projection operator $\mathscr{P}_\alpha[\mathbf{x}]$

Initialize:
- $\mathbf{X}(\omega) \leftarrow \mathbf{x}$ (transform to frequency domain)

for $\omega = \omega_{min} : \omega_{max}$ **do**
- $[\mathbf{U}(\omega), \mathbf{S}(\omega), \mathbf{V}(\omega)] = \text{svd}[\mathbf{X}(\omega)]$
- $\hat{S}_{l,l}(\omega) = \max[S_{l,l}(\omega) - \alpha, 0]$
- $\hat{\mathbf{X}}(\omega) = \mathbf{U}(\omega)\hat{\mathbf{S}}(\omega)\mathbf{V}^*(\omega)$

end for

$\mathbf{d}^k \leftarrow \hat{\mathbf{X}}(\omega)$ (transform back to time)

EXAMPLE

We test the proposed algorithm with a synthetic example simulated from the *zz* component of a 3D-9C synthetic VSP data set. The geometry of sources and receivers are portrayed in Figure (1). The dataset contains a regular grid of 205 × 205 sources. The x and y source intervals are 16.67m. A group of 31 detectors are deployed downhole right in the center from a depth of 1350 m to 1850 m. In our simultaneous source acquisition, we assume a rather unrealistic scheme with one vessel moving and firing at short time intervals. A more realistic scenario requires more than one vessel firing at random time intervals. The time interval between two adjacent sources follows a uniform distribution. Figure (2) displays the total acquisition time for a regular survey in conjunction with the blended survey modeled in our example. The acquisition time was compressed to 50% of the conventional acquisition.

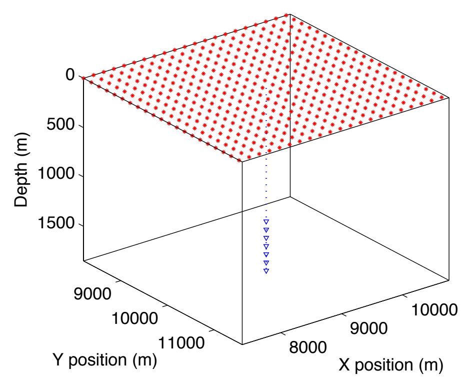

Figure 1: Distribution of sources and receivers. Each red point represents a group of 10 sources and each blue triangle represents 5 receivers.

We apply the proposed algorithm to deblend the synthetic VSP data. Figure (3) shows the distribution of singular values for one ideal (pre-blending) common receiver gather at 25Hz (red), singular values at the same frequency for the pseudo-deblended gather (blue) and the deblended common receiver gather (green) also at 25 Hz. It is clear that the incoherent noise in the pseudo-deblended data increased the rank of the original data. The proposed algorithm has eliminated the noise and re-established a distribution of singular values similar to the distribution of

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singular values of the ideal data. Figure (4) shows a time slice of the results after 25 iterations. Figure (5) shows the deblending result for the center shot and Figure (6) shows the result for the center receiver. The interferences from simultaneously fired shots are effectively suppressed. We improve the signal to noise ratio of the pseudo-deblended dataset from 0 dB to 27.8 dB. As a result, the unblended solution becomes comparable with the true shot record.

CONCLUSION

This article proposes a deblending algorithm based on nuclear norm regularization. The method operates directly on multidimensional data and relies on the coherency of the desired signal in common receiver gathers. The steepest descent method has been utilized in conjunction with a soft thresholding operator applied to the singular values to enforce solutions with minimal nuclear norm. Through tests with synthetic examples, we show that the interferences can be effectively suppressed by the proposed method. The proposed method can be further applied to the joint reconstruction (interpolation) and separation of simultaneous source data.

In the current algorithm, a rank constraint is applied to matrices in the frequency domain. We are also investigating expanding the algorithm to tensors (Kreimer et al., 2013) and, therefore, allow to deblend multiple receivers at one time.

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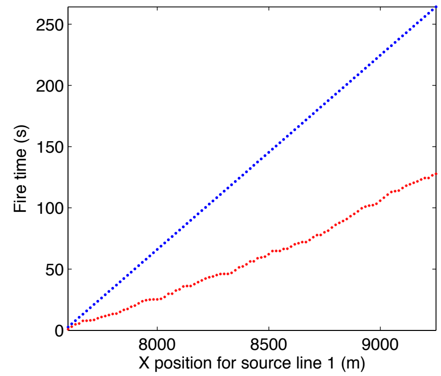

Figure 2: Distribution of firing time of the first 100 shots for conventional (blue) and simultaneous source acquisition (red).

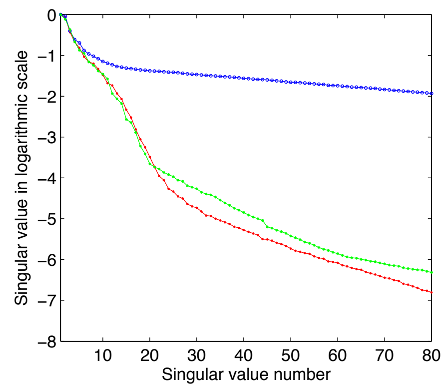

Figure 3: Distribution of the first 80 singular values in logarithmic scale for the center common receiver gathers at 25 Hz: the distribution of singular values for one ideal (pre-blending) common receiver gather (red), the pseudo-deblended gather (blue) and the deblended common receiver gather (green).

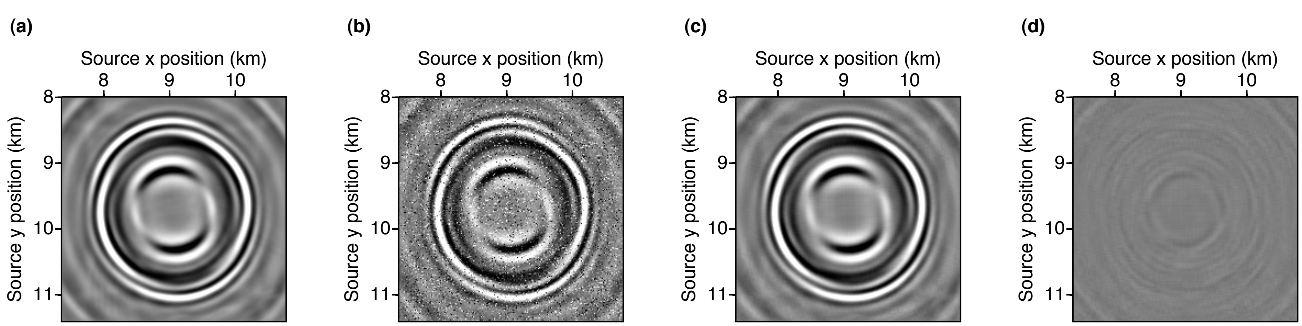

Figure 4: Dual domain minimum nuclear norm deblending results. (a) The real unblended time slice at 1.2 s. (b) Pseudo-deblended time slice. (c) Deblended time slice after after 25 iterations of the proposed algorithm. (d) Differences between (a) and (c). In this example, the signal-to-noise ratio after separation is 27.78dB.

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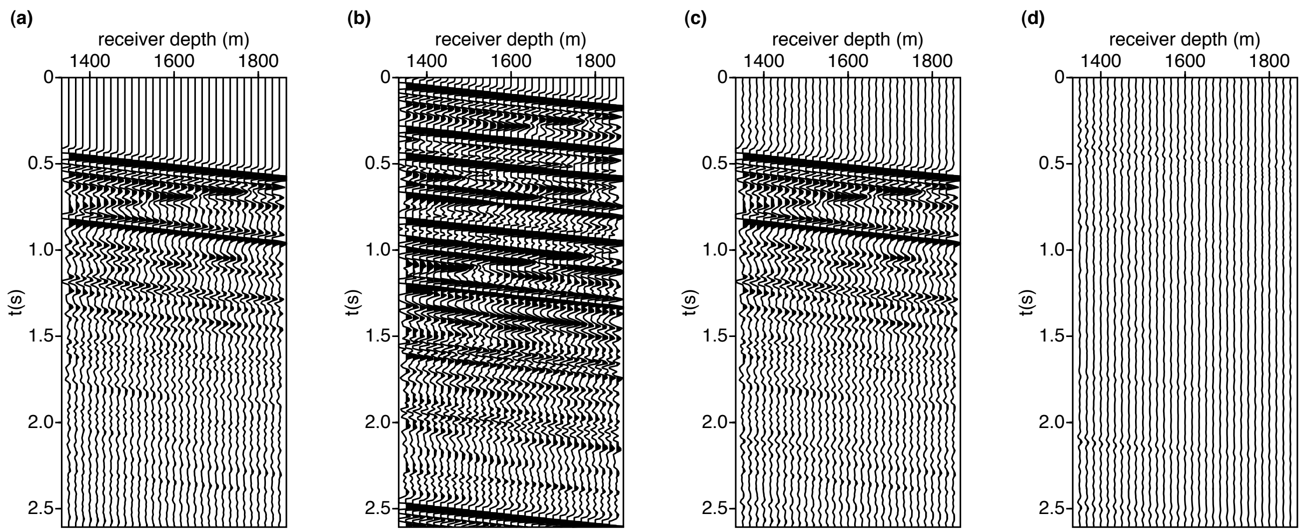

Figure 5: Results of separation via dual domain minimum nuclear norm deblending algorithm: (a) The real unblended common shot gather (center shot). (b) Pseudo-deblended shot record. (c) Deblended shot record after 25 iterations. (d) Differences between (a) and (c).

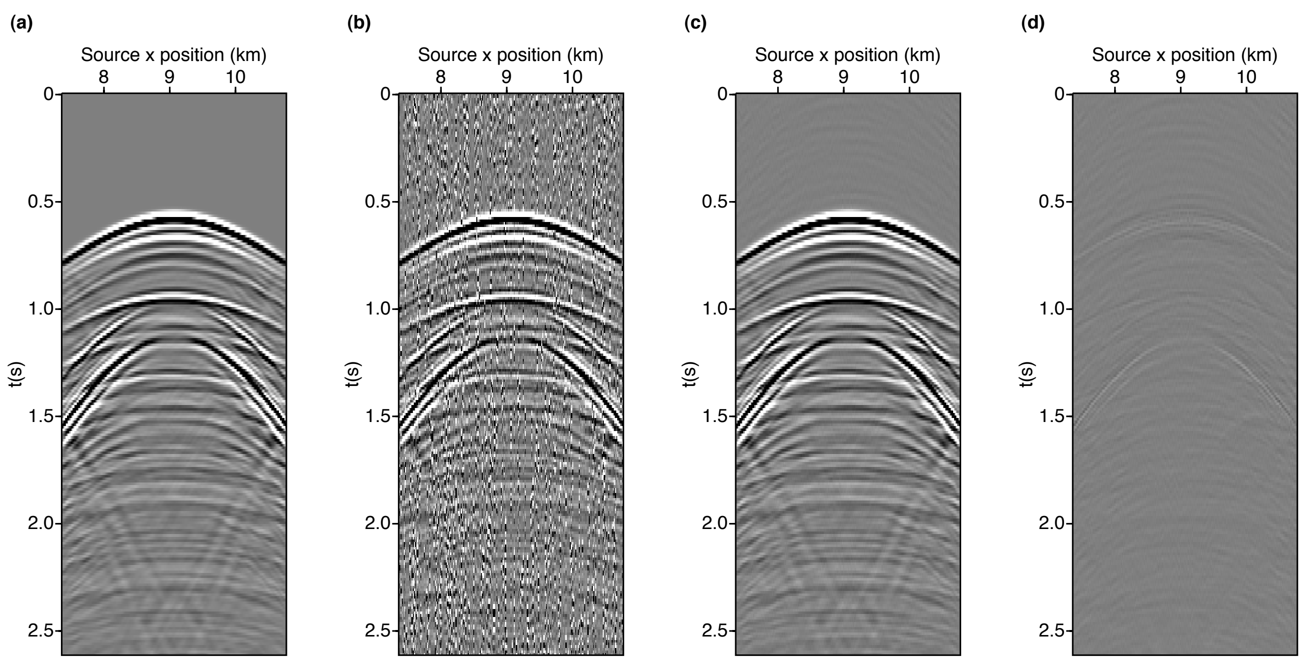

Figure 6: Results of separation via dual domain minimum nuclear norm deblending algorithm: (a) The real unblended common receiver gather (center receiver). (b) Pseudo-deblended common receiver gather. (c) Deblended common receiver gather after 25 iterations. (d) Differences between (a) and (c).

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EDITED REFERENCES

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