Geophysical Prospecting, 2006, **54**, 487–488

Comment on 'Non-minimum-phase wavelet estimation using second-and third-order moments' by Wenkai Lu

Danilo R. Velis[1*] and Mauricio D. Sacchi[2]

[1]*Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n, La Plata, Buenos Aires, B1900FWA, Argentina,* [2]*University of Alberta, Department of Physics, Avadh Bhatia Physics Laboratory, Edmonton, Alberta T6G 2J1, Canada*

Received November 2005, revision accepted December 2005

The work of Lu (2005) deals with one of the key problems in seismic data processing: wavelet estimation. As is usual, the convolutional model is assumed: the seismic trace x_t is equal to the convolution of the wavelet b_t with the reflectivity series s_t, plus Gaussian noise. Lu (2005) proposes a method that utilizes the fourth-order moment (FOM) of the seismic trace to estimate the third-order moment (TOM) of the wavelet, from which its phase is extracted after a few operations that involve two deconvolutions via spectral divisions. The first deconvolution (which is 2D) is carried out to estimate the reflectivity TOM, which in turn is used to estimate a scaled and shifted version of the actual reflectivity. The second deconvolution (which is 1D) is carried out to estimate the wavelet phase. The amplitude spectrum of the wavelet is estimated using the autocorrelation of the data.

The wavelet TOM, which is the key of the proposed wavelet estimation method (step 1), is estimated from its FOM, which in turn is estimated from the seismic trace FOM using Lu's equations (6) and (23), respectively. However, according to the definition of the higher-order moment functions given in Lu's equation (1), equation (23) is not correct. It can be proved that, provided the reflectivity is an independent, identically distributed (i.i.d) and non-Gaussian process, the fourth-order cumulant of the seismic trace equals, within a scale factor, the fourth-order moment of the wavelet, i.e.

$$m_4^b(\tau_1, \tau_2, \tau_3) \propto c_4^x(\tau_1, \tau_2, \tau_3) \neq m_4^x(\tau_1, \tau_2, \tau_3), \qquad (1)$$

where $c_4^x(\tau_1, \tau_2, \tau_3)$ is the trace fourth-order cumulant (FOC). The same proportionality holds for third- and second-order statistics (Mendel 1991; Nikias and Mendel 1993). In this sense, and ignoring the smoothing window, wavelet and trace FOMs are not equivalent, as expressed by equation (23). In any case, it is not clear why it is necessary to rely on fourth-order statistics to estimate the wavelet TOM, when it is possible to use third-order statistics, which exhibit less variability. Instead, the fact can be used that

$$m_3^b(\tau_1, \tau_2) \propto c_3^x(\tau_1, \tau_2) = m_3^x(\tau_1, \tau_2), \qquad (2)$$

for a zero-mean process. This would make it unnecessary to use equation (6) to derive the wavelet TOM from its FOM (here a 2D Parzen window can be used to smooth and improve the wavelet TOM estimate). At this point it is important to note that equation (6) is usable only for non-zero mean processes. Seismic wavelets and traces are inherently zero-mean processes because of the recording systems used in seismic exploration; thus, the proposed method to estimate the wavelet TOM via equation (6) would be very unstable. The position would become worse when dealing with the spectral division of equation (12) under noisy conditions.

Steps 2–6 of the proposed method are carried out to estimate the wavelet (and the reflectivity) from the wavelet and trace TOMs. Equation (19) provides a direct method to estimate a scaled and shifted version of the reflectivity from the maximum time-delay slice (MTDS) of its TOM (obtained after the above-mentioned spectral division). Following another spectral division, the wavelet phase is estimated.

Again, we believe that some steps are unnecessary. The wavelet can be estimated directly from its TOM, without the need to estimate any reflectivity TOM or MTDS whatsoever (steps 2 and 3 of the proposed method), using the C(q,k) formula (Mendel 1991), which also uses a single slice of the TOM, and leads to

$$\hat{b}_t = \frac{\hat{m}_3^b(q, t)}{\hat{m}_3^b(-q, -q)} = \frac{\hat{m}_3^b(q, t)}{\hat{m}_3^b(q, 0)}, \qquad t = 1, 2, \cdots, q. \qquad (3)$$

*E-mail: velis@fcaglp.unlp.edu.ar

There are also some equivalent formulae based on fourth-order statistics.

However, direct methods based on a single slice of higher-order covariance or moment functions do not provide any filtering to reduce the effects of the errors derived from the fact that these functions are simple estimates, thus the above equation is not practical from a computational point of view (Mendel 1991). The same argument applies to the estimation of the reflectivity using the MTDS.

REFERENCES

Lu W. 2005. Non-minimum-phase wavelet estimation using second- and third-order moments. *Geophysical Prospecting* **53**, 149–158.

Mendel J. 1991. Tutorial on higher-order statistics in signal processing and system theory: Theoretical results and some applications. *Proceedings of the IEEE* **79**, 278–305.

Nikias C.L. and Mendel J.M. 1993. Signal processing with higher-order spectra. *IEEE Signal Processing Magazine* **10**, 10–37.